Year 12 **Topic 2** **Trigonometric Functions and Graphs** **Time: 3 Weeks**

TOPIC: Trigonometric Functions **Syllabus pages: 51-52** **Sub Topic: Trigonometric Functions and Graphs MA – T3**			
Focus: The principal focus of this subtopic is to explore the key features of the graphs of trigonometric functions and to understand and use basic transformations to solve trigonometric equations. Students develop an understanding of the way that graphs of trigonometric functions change when the functions are altered in a systematic way. This is important in understanding how mathematical models of real-world phenomena can be developed.			
Student Outcomes: MA12 – 1, 5, 9, 10 A student: › uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts MA12-1 › applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs MA12-5 › chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9 › constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10			
	Student is able to:	Implications, considerations and implementations	Resources
(i)	Examine and apply transformations to sketch functions of the form $y = kf(a(x + b)) + c$, where a, b, c and k are constants, in a variety of contexts, where $f(x)$ is one of $\sin x$, $\cos x$ or $\tan x$. -State the domain and range. -State the period and amplitude	- use technology or otherwise to examine the effect on the graphs of changing the **amplitude** $y = kf(x)$, the **period,** $y = f(ax)$, the **phase,** $y = f(x + b)$, and the **vertical shift,** $y = f(x) + c$ - use k, a, b, c to describe transformational shifts and sketch graphs. {Pupils should be lead to the fact that the amplitude is $\lvert a \rvert$ and period is $\frac{2\pi}{n}$ for $y = a \sin nx$ }. Graph functions of the form:	**Other resources:** Video: Transforming trigonometric graphs with Desmos, here. Desmos file used in the video is here. Worksheet: How to draw trigonometric graphs, here, with a teacher PowerPoint, here **From Mathematics 2 Unit / Advanced:** 2017-14a, 2016-6, 2016-8, 2013-6, 2010-8c, 2006-7b(i)(ii), 2001-4c(i), 2000-6a, 1996-7a, 1996-10a(i) **From Extension 1:** 2015-10

		$y = a\sin bx$, $y = a\sin bx + c$, $y = a\sin(bx+\varepsilon)$, etc. They should graph $y = a\sin(bx+\varepsilon)$ by suitably transforming the axes for $y = \sin x$. e.g. $y = \sin 2x$, $y = 4\cos 3x$, $y = \frac{1}{2}\sin(\frac{\pi}{2}x)$, $y = 5 + 3\cos x$, $y = 3 - \sin 2x$. NOTE: $y = 4\cos(2x - \pi)$ should be graphed as $y = 4\cos 2(x - \frac{\pi}{2})$ with a period of π and a phase shift of $\frac{\pi}{2}$ to the right.	
(ii)	Solve trigonometric equations involving functions of the form: $kf(a(x+b)) + c = 0$, using technology or otherwise, within a specified domain (in degrees and radians).	e.g. Solve for $0 \le x \le 2\pi$ (i) $2\sin x = 1$ (ii) $3\cos^2 x + 2\sin x - 2 = 0$ (iii) $\cos x = \sec x$	**Other resources:** Worksheet: How to solve trigonometric equations, here, with a teacher PowerPoint, here. **From Mathematics 2 Unit / Advanced:** 2016-11g, 2015-12a, 2014-7, 2012-6, 2011-2b, 2009-1e, 2007-4a, 2007-7b(i), 2005-2a, 1999-10a **From Extension 1:** 2009-3b(i)(ii)
(iii)	Use trigonometric functions of the form $g(x) = kf(a(x+b)) + c$ to model and/or solve practical problems involving periodic phenomena.		**Other resources:** An example of a periodic phenomenon which can be modelled using a sine curve is the rising and falling of the tide in a body of water such as Sydney Harbour. This concept could possibly used as the basis for the investigative-style assignment. • A website in which predicted tides are shown in a graph is here. • This website compares predicted tides to actual tides, here and here. • Videos: How do tides work, here and here. • Article about tsunamis, including one which reached Sydney, here. • The graph in the previous article would have been drawn with equipment like this.

			• Two HSC questions which could be used as investigations: 2016-13a, 2004-7a Another example is time of sunrise/sunset and number of daylight hours of each day during the year, here. **From Mathematics 2 Unit / Advanced:** 2018-15a, 2013-13a, 2009-7b, 2002-8b **From Extension 1:** 2016-13a, 2004-7a, 1997-3a
(iv) ME	Sketch Inverse Trigonometric Graphs	These graphs to include ones where the graph has been translated vertically and horizontally. e.g. $y = 2\sin^{-1} 3x$, $y = \pi \cos^{-1} \pi x$, $y = 2\tan^{-1}\left(\frac{x}{4}\right)$, $y = \frac{\pi}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)$. See Graphs of Inverse Functions Harder functions include: e.g. $y = \sin^{-1} x + \cos^{-1} x$, $y = \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ where $x \neq 0$, $y = 2\sin^{-1}\sqrt{x} + \cos^{-1}(2x - 1$. Investigate the fact that $\cos^{-1}(\cos x) \neq x$ (a) Graph $y = \cos^{-1}(\cos x)$ by plotting points to see behaviour of this function. (b) Graph $y = \cos^{-1}(\cos x)$ and corresponding graphs for $y = \sin^{-1}(\sin x)$ and $y = \tan^{-1}(\tan x)$ See Harder Graphing and Harder Graphing Solutions	