

3rd Grade Unit 5 Priorities Document

Please see the [Draft Grade 3 Math Overview and Scope and Sequence](#) for important information about the year and emphases for each unit.

Overarching Big Ideas

- *Less is more*
- *Depth vs. breadth*
- *Relationships over everything*
- *Access for all, especially emerging bilinguals & students with disabilities*

	Window (change)	Big Idea	Comments	Standards (Priority Standards bolded)
Unit 3.5 Expanding Multiplication	4 weeks (was 15 days)	Relationships between numbers in multiplication can be described using arithmetic patterns and interpreted and expressed in multiple ways by applying properties of operations and mathematical strategies.	<ul style="list-style-type: none"> • Incorporate foundational work with addition strategies within 20 and within 100 (from 2.3) to support using the distributive property. • Reduce emphasis on arithmetic patterns 	3.OA.1 3.OA.3 3.OA.5 3.OA.9

Norms



Answers are important, but they are not the math. 	Talk about each other's thinking. 	Errors are gifts that promote discussion. 	Ask questions until ideas make sense. 	Use multiple strategies and multiple representations. 
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2020-21 SFUSD Elementary Math Distance Learning Resources	Gr 3 Curriculum Portal	Games in the Gr 3 Curriculum	Math Talks Bank
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Optional Routines:	<ul style="list-style-type: none"> • Number of Days in School (Spanish) • Counting Routine • Data Routine • Number of the Day Routine / Jamboard Number of the Day (Spanish). 	Optional Math Talks (Spanish) include Shape Talks and Which One Doesn't Belong
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[3.5 Slidedeck](#) / [Spanish](#) (translations pending)



	New Learning	Re-engagement	Other Resources
	<ul style="list-style-type: none"> Several different patterns can be observed in the multiples of numbers. These patterns can help build towards fluency with multiplication. <ul style="list-style-type: none"> When we multiply a number by 1, the product is that number. When we multiply a number by 0, the product is zero. When we multiply two numbers, we can separate one of them into parts, multiply each part by the other number, and then add the products. The multiplication chart is a useful tool for organizing products and looking for patterns. Understanding patterns in multiplication helps to find unknown factors and products. 	<ul style="list-style-type: none"> Multiplication is a way to find out how many objects you have altogether when those objects come in equal groups. Multiplication expressions can be represented in a variety of ways, including: Circles and Stars, Arrays, Number Lines, and Tape Diagrams We can use our understanding of place value and sums to ten to add within twenty (from Grade 2) 	<p>3.5 Podcast Unit 3.5 Family Letter S C * 3.5 Tech Resources</p> <p>* S = Spanish Student Page * C = Chinese Student Page</p>
			<p>Classwork and Homework PDFs</p> <p>3.5 Classwork S C 3.5 Homework S C</p>
CCSS Math Content Standards:	<p>Operations and Algebraic Thinking Represent and solve problems involving multiplication and division. 3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as 5×7.</i> 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p>Understand properties of multiplication and the relationship between multiplication and division. 3.OA.5 Apply properties of operations as strategies to multiply and divide.* <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8(5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i> * Students need not use formal terms for these properties.</p> <p>Solve problems involving the four operations, and identify and explain patterns in arithmetic. 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i></p>		
Unit Warm-up: Connect the topic to something familiar to your	<p>What Comes in Groups?</p> <ul style="list-style-type: none"> Remind students of things on a dog that come in groups (4 legs, 2 ears, etc.) Brainstorm things that come in groups in the world. Work on: Jamboard: Unit Warm-up: Things that come in Groups (Spanish) 		

students.	<ul style="list-style-type: none"> Slide deck: What Comes in Groups (Spanish) <p>The ideas generated in this warm-up will be returned to throughout the unit.</p>		
	Recommended Lessons	Strengths to highlight & Notebook Prompt	Notes and Continuing Activities
Lesson 1 (3.5 Entry Task) Standard(s) 3.OA.1	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Review ways of representing multiplication (from Unit 3.3) <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> This card sort is not in the Classwork books. Multiplication Cards FAL BLM S C 3.5 Student Slide Deck (Spanish) individual or group work Grade 3 Unit 5 Lesson 1 Card Sort Jamboard (English & Spanish) individual or group work Seesaw 3.5 Lesson 1 Card Match (Spanish) - for individual work <p>SUMMARIZE Whole Class or Groups:</p> <p>Core Math to Emphasize</p> <ul style="list-style-type: none"> Multiplication is a way to find out how many objects you have altogether when those objects come in equal groups. Multiplication expressions can be represented in a variety of ways, including: Circles and Stars, Arrays, Number Lines, and Tape Diagrams 	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that there are different ways to represent a multiplication expression. <p> How can you represent 3×2? Can you represent it a different way?</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>Options for Continuing Activities</p> <ul style="list-style-type: none"> Any of the Optional Unit Resources from Unit 3.3 might be useful continuing activities in this unit. The focus in Unit 3.3 and this unit is to develop student's conceptual understanding of multiplication - understanding multiplication as equal groups, arrays, and measurement. As this understanding is developing, students can also begin to develop their procedural fluency with multiplication. We recommend this online activity: Multiplication by Heart which incorporates visuals and uses subitizing to connect different ways of representing multiplication to fluency. Deck 1: repeated addition. Deck 2: arrays. Deck 3: based on prime factors.
Lesson 2 (3.5 LS2 Day 1) Standard(s) 3.OA.1 and 3.OA.9	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm What does it mean to count down? Introduce task <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> The 3.5 Student Slide Deck (Spanish) for individual or group work Seesaw 3.5 Lesson 2 Circles and Stars Countdown (Spanish) - for individual work Classwork books: Plates and Counters Countdown Part 1 S C & Plates and Counters Countdown Part 2 S C - (note that these use "Plates and Counters" language instead of Circles and Stars) - for individual work. 	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that visual patterns can help us understand ideas. <p> What patterns did you notice today?</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	

	<p>SUMMARIZE Whole Class or Groups: Core Math to Emphasize</p> <ul style="list-style-type: none"> There is a pattern we see when we multiply a number by 1. The product is that number. There is a pattern we see when we multiply a number by 0. The product is zero. 		
<p>Lesson 3 (3.5 LS2 Day 2)</p> <p>Standard(s) 3.OA.1 and 3.OA.9</p>	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Introduce task with Circles and Stars and Skip counting <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> The BLM Multiplication Patterns BLM S C is not in the Classwork books. 3.5 Student Slide Deck (Spanish) for individual or group work <ul style="list-style-type: none"> This video: Making Multiplication Stars has instructions for using the slides. Seesaw 3.5 Lesson 3 Multiplication Stars (Spanish) - for individual work <p>SUMMARIZE Whole Class or Groups: Core Math to Emphasize</p> <ul style="list-style-type: none"> Several different patterns can be observed in the multiples of numbers. These patterns can help build towards fluency with multiplication. 	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that we can find and explain patterns in math that help us understand ideas. <p> Our class of mathematicians knows that we can find and explain patterns in math that help us understand ideas.</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>Options for Continuing Activities</p> <ul style="list-style-type: none"> This activity could potentially extend over a number of days, as each star may take students a while to create. The All Star Patterns (Spanish) slide deck includes all the numbers 1-10. Optional discussions include comparing all the stars and comparing all the 100 charts. Wheel Math is an iPhone / iPad app for making stars like the ones made in this lesson.
<p>Lesson 4 - Part 1 of using the Distributive Property to multiply large numbers, adapted from 3.5 LS1 Day 1)</p> <p>Standard(s) 3.OA.5</p>	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Introduce activity using a familiar situation: Rahima's Dogs <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> This page is not in the Classwork books. There is an optional CW page here: 3.5 Distance Learning Lesson 4 Large Numbers (Spanish) 3.5 Student Slide Deck (Spanish) has slides for individual or group work Seesaw 3.5 Lesson 4 Large Numbers (Spanish) - for individual work <p>SUMMARIZE Whole Class or Groups:</p>	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that we can organize information in math to help us understand it. <p> How can you think about multiplying large numbers like 8 x 3?</p>	<p>Options for Continuing Activities</p> <ul style="list-style-type: none"> Another online app that begins to build multiplication fluency with visuals is Bunny Times. This interactive activity uses carrots in a field to represent the array model of multiplication. Students can apply strategies such as skip counting, adding on, and deconstructing

	<p>Core Math to Emphasize</p> <ul style="list-style-type: none"> When we multiply two numbers, we can separate one of them into parts, multiply each part by the other number, and then add the products. 	<p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>problems like 7×8 into $(5 \times 8) + (2 \times 8)$.</p>
<p>Lesson 5 - Part 2 of using the Distributive Property to multiply large numbers, adapted from 3.5 LS1 Day 1)</p> <p>Standard(s) 3.OA.5</p>	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Review dice patterns and introduce task <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> Optional CW page here: 3.5 Multiplying with Groups of Dots (Spanish) 3.5 Student Slide Deck (Spanish) for individual or group work Seesaw 3.5 Lesson 5 Multiplying with Groups of Dots (Spanish) - can be used for individual work <p>SUMMARIZE Whole Class or Groups:</p> <p>Core Math to Emphasize</p> <ul style="list-style-type: none"> When we multiply two numbers, we can separate one of them into parts, multiply each part by the other number, and then add the products. 	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that asking questions can help us understand new ideas. <p> Our class of mathematicians knows that asking questions can help us understand new ideas.</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>Options for Continuing Activities</p> <p>This lesson uses <i>subitizing</i>* to develop student's understanding of multiplying larger numbers and promote fluency.</p> <ul style="list-style-type: none"> This Multiplication Subitizing Slide Deck** for individuals or small groups to continue to practice this skill. There is also a PDF of Subitizing cards that could be useful if you (or students) have a printer. This video shows the cards in use.
<p>*<i>Subitizing</i> is the ability to instantly recognize 'how many', as in the 5 dots on a die. To learn more about subitizing and multiplication see Graham Fletcher's blog or The Recovering Traditionalist.</p> <p>**Slide deck created by Jennifer H. inspired by Graham Fletcher's Blog Post</p>			
<p>Lesson 6 (3.5 LS 1 Day 4 Part 1)</p> <p>Standard(s) 3.OA.1 & 3.OA.9</p>	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Remind students of multiplication charts and introduce activity <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> Student CW page here: Multiplication Chart 1 to 10 S C (note that this chart does not include the zero) 3.5 Student Slide Deck (Spanish) individual or group work Seesaw 3.5 Lesson 6 and 7 Multiplication Chart (Spanish) for individual work - NOTE: If students are working in Seesaw, have them save their work as a draft so they can return to it later for lesson 7. 	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that charts can help us organize our ideas and see patterns. <p> Explain one of the patterns you saw or used today.</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>Options for Continuing Activities</p> <ul style="list-style-type: none"> In coming lessons, students will continue to investigate patterns in the multiplication chart.

	<p>SUMMARIZE Whole Class or Groups: Core Math to Emphasize</p> <ul style="list-style-type: none"> The multiplication chart is a useful tool for organizing products and looking for patterns. 		
<p>Lesson 7 (3.5 LS 1 Day 4 Part 2)</p> <p>Standard(s) 3.OA.5 & 3.OA.9</p>	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Review “easy” products and distributive property to launch activity <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> Student CW page here: Multiplication Chart 1 to 10 S C (note that this chart does not include the zero) 3.5 Student Slide Deck (Spanish) for individual or group work Seesaw 3.5 Lesson 6 and 7 Multiplication Chart (Spanish) for individual work <p>SUMMARIZE Whole Class or Groups: Core Math to Emphasize</p> <ul style="list-style-type: none"> Unknown multiplication facts can be found by using known facts. The multiplication chart is a useful tool for organizing products and looking for patterns. 	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that charts can help us organize our ideas and see patterns. <p> Explain one of the patterns you saw or used today.</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	
<p>Lesson 8 (3.5 LS2 Day 3)</p> <p>Standard(s) 3.OA.1 & 3.OA.9</p>	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Review odd and even numbers to introduce activity <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> Multiplication Chart 0 to 10 S C Multiplication Chart 0 to 10 Filled In BLM S C (not in the student booklets) Patterns in Multiplication S C for individual work 3.5 Student Slide Deck (Spanish) for individual or group work Seesaw 3.5 Lesson 8 Patterns in the Multiplication Chart (Spanish) for individual work <p>SUMMARIZE Whole Class or Groups: Core Math to Emphasize</p> <ul style="list-style-type: none"> The Multiplication Chart has patterns that help us understand multiplication and develop fluency. 	<p>Strengths:</p> <ul style="list-style-type: none"> When we organize information we can see patterns that help us understand ideas. <p> Explain one of the patterns you saw or used today.</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>Options for Continuing Activities</p> <ul style="list-style-type: none"> This interactive multiplication chart has lots of options for students to color-code products. Interactive Multiplication Tables provides a large multiplication table that allows students to make connections, spot patterns and improve their number sense. The i button in the upper right gives guidance on how to use this app.

	<ul style="list-style-type: none"> Even factors always produce even products. Odd factors only produce odd products if multiplied by an odd factor. 		<ul style="list-style-type: none"> Students can continue to develop their multiplication fluency with activities from previous lessons.
Lesson 9 (3.5 Expert Task) Standard(s) 3.OA.1 & 3.OA.9	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Introduce the task <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> Student CW page here: The Wheel Shop S C 3.5 Student Slide Deck (Spanish) for individual or group work Seesaw 3.5 Lesson 9 The Wheel Shop (Spanish) for individual work <p>SUMMARIZE Whole Class or Groups:</p> <p>Core Math to Emphasize</p> <ul style="list-style-type: none"> Understanding patterns in multiplication helps to find unknown factors and products. 	<p>Strengths:</p> <ul style="list-style-type: none"> We can solve problems by trying different solutions. <p> How many tricycles could be in the shop if there were an odd number of wheels?</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>Options for Continuing Activities</p> <ul style="list-style-type: none"> Students can continue to develop their fluency with any of the previously suggested resources.
Lesson 10 (Milestone Task) Standard(s) 3.OA.1, 3.OA.3, 3.OA.5, & 3.OA.9	<p>LAUNCH Whole Class or Groups:</p> <ul style="list-style-type: none"> Pick a focus norm Introduce the task <p>EXPLORE Independent or Group work:</p> <ul style="list-style-type: none"> 3.5 Student Slide Deck (Spanish) for individual or group work 3.5 Distance Learning Milestone Task (Spanish) Seesaw pt 1: 3.5 Milestone Writing Multiplication Stories (Spanish) Seesaw pt.2: 3.5 Milestone Patterns on the Multiplication Chart (Spanish) Seesaw pt. 3: 3.5 Milestone Breaking up Factors (Spanish) <p>SUMMARIZE Whole Class or Groups:</p> <p>Core Math to Emphasize</p> <ul style="list-style-type: none"> Multiplication is a way to find out how many objects you have altogether when those objects come in equal groups. There are a variety of ways to solve multiplication problems. The multiplication chart is a useful tool for organizing products and looking for patterns. 	<p>Strengths:</p> <ul style="list-style-type: none"> Our class of mathematicians knows that mistakes help us learn. <p> One new thing I learned about multiplication in this unit is:</p> <p>The optional Notebook Prompts may be completed on this Math Notebook or this Math Notebook</p>	<p>Options for Continuing Activities</p> <ul style="list-style-type: none"> Students can continue to develop their fluency with any of the previously suggested resources. <p>Milestone Scoring</p> <ul style="list-style-type: none"> Use the 3.5 Distance Learning Milestone Task Rubric as guidance to review student work The Milestone Task is designed to give you information about what your students know and are able to do. This will help you plan for unit 3.6, the final multiplication unit.

	<ul style="list-style-type: none"> When we multiply two numbers, we can separate one of them into parts, multiply each part by the other number, and then add the products. 		<ul style="list-style-type: none"> Consider giving students the opportunity to revise their work after the summary. Milestone scores can be entered in illuminate using these instructions.
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Grade 3 - Focus on Multiplication

In Grade 3, Multiplication and Division are introduced. This topic is so important, that close to ½ of the year is devoted to its study.

Units 3.3, 3.5, and 3.6 are the 3 units devoted to multiplication. Unit 3.7 is devoted to division, and unit 3.9, Area Measurement, focuses largely on multiplication.

How multiplication is developed across the 3 key multiplication units:

	Unit 3.3	Unit 3.5	Unit 3.6																																																																																																																																																
All units focus on <ul style="list-style-type: none">• Understanding the meaning of the multiplication operation• Writing multiplication expressions and equations to represent multiplicative situations• Developing fluency with multiplication calculations																																																																																																																																																			
Numbers used:	Focus on groups of 1, 2, 3, 4, and 5.	Expand to groups of 6, 7, 8, and 9, followed by 0 and 10.	Focus on groups of 10.																																																																																																																																																
Problems types focus:	Equal groups: (# of groups) x (group size) = total. Measurement (Number Line) Beginning arrays.	Equal groups: (# of groups) x (group size) = total. Measurement Extensive use of arrays.	Equal groups, measurement, and arrays. See the CCSS Math Glossary for more information on problem types.																																																																																																																																																
New Learning:	Meaning of multiplication - situations involving equal groups, measurement (number line), and arrays. Expressions & equations. Understanding and using the Commutative Property.	Combining known facts to find unknown facts using the Distributive Property of Multiplication over Addition, and looking for patterns in the multiplication table.	Apply the Associative Property of Multiplication and begin to apply place value understanding to multiplication																																																																																																																																																
Multiplication table:	Begin to build groups of equations: 1 x 5 = 5 2 x 5 = 10 3 x 5 = 15 4 x 5 = 20 Etc.	<table><tr><th>x</th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr><tr><th>0</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><th>1</th><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><th>2</th><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr><tr><th>3</th><td>0</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>21</td><td>24</td><td>27</td><td>30</td></tr><tr><th>4</th><td>0</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr><tr><th>5</th><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td><td>45</td><td>50</td></tr><tr><th>6</th><td>0</td><td>6</td><td>12</td><td>18</td><td>24</td><td>30</td><td>36</td><td>42</td><td>48</td><td>54</td><td>60</td></tr><tr><th>7</th><td>0</td><td>7</td><td>14</td><td>21</td><td>28</td><td>35</td><td>42</td><td>49</td><td>56</td><td>63</td><td>70</td></tr><tr><th>8</th><td>0</td><td>8</td><td>16</td><td>24</td><td>32</td><td>40</td><td>48</td><td>56</td><td>64</td><td>72</td><td>80</td></tr><tr><th>9</th><td>0</td><td>9</td><td>18</td><td>27</td><td>36</td><td>45</td><td>54</td><td>63</td><td>72</td><td>81</td><td>90</td></tr><tr><th>10</th><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td></tr></table> <p>Continue building table and look at patterns:</p>	x	0	1	2	3	4	5	6	7	8	9	10	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2	3	4	5	6	7	8	9	10	2	0	2	4	6	8	10	12	14	16	18	20	3	0	3	6	9	12	15	18	21	24	27	30	4	0	4	8	12	16	20	24	28	32	36	40	5	0	5	10	15	20	25	30	35	40	45	50	6	0	6	12	18	24	30	36	42	48	54	60	7	0	7	14	21	28	35	42	49	56	63	70	8	0	8	16	24	32	40	48	56	64	72	80	9	0	9	18	27	36	45	54	63	72	81	90	10	0	10	20	30	40	50	60	70	80	90	100	Extend table and pattern work to multiples of 10.
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10	0	10	20	30	40	50	60	70	80	90	100																																																																																																																																								

Multiplication Fluency through Properties and Patterns

NCTM defines procedural fluency as “the ability to apply procedures accurately, efficiently, and flexibly.” This position paper describes the development of procedural fluency as building from “initial exploration and discussion of number concepts to using informal reasoning strategies and the properties of operations to develop general methods for solving problems.”



Fluency is not memorization. Like fluency with a language, it is an ability to *use* numbers sensibly and efficiently. Fluency is attained through a combination of number sense (the ability to use numbers flexibly), understanding numerical relationships, and thinking about strategies. For more on this topic, please see *Fluency Without Fear* - tinyurl.com/ychq9mds

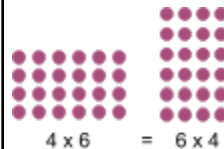
While Unit 3.3 exposed students to what multiplication is and how it can be represented and solved, Unit 3.5 is intended to help students develop fluency with multiplication. This is done through multiple experiences that build their number sense and give them opportunities to investigate:

- Patterns in multiplication
- How the properties of multiplication (especially the Distributive property) can be used strategically.

Students learn new multiplication facts by building on already known or easily computed facts (such as multiplication by 1s, 2s, and 5s). As the numbers increase in magnitude, the strategies of counting all and skip counting / double counting become less efficient, and students should increasingly rely on using their knowledge of the properties and patterns of multiplication.

In this unit, students develop fluency as they learn about:

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100



$$4 \times 6 = 6 \times 4$$

The **Commutative property of multiplication** was introduced in unit 3.3
If $6 \times 4 = 24$ is known,
then $4 \times 6 = 24$ is also.



$$8 \times 7 = (8 \times 5) + (8 \times 2)$$

The **Distributive property of Multiplication over Addition:**
I can find 8×7 by splitting 7 into 5 + 2:
 $8 \times 7 = 8 \times (5 + 2) =$
 $(8 \times 5) + (8 \times 2) = 40 + 16 = 56.$

Students do not need to know the names of these properties, nor be able to state them algebraically. But they should see, through experience, that they hold for multiplication. They should be able to explain that $3 \times 5 = 5 \times 3$ and state that *the order in which the factors are multiplied doesn't change the product.*



8 x 7 is even

$$\begin{array}{r} 1 \times 5 = 5 \\ 2 \times 5 = 10 \\ 3 \times 5 = 15 \\ 4 \times 5 = 20 \\ 5 \times 5 = 25 \end{array}$$

$$\begin{array}{r} \times 5 = \\ \underline{ 5} \text{ or } \underline{ 0} \end{array}$$

Patterns in multiplication:

e.g.: 7×8 must be even because 7 groups of 8 is 7 groups of even numbers, which would be even.

A number multiplied by 5 always has a 0 or 5 in the ones place.