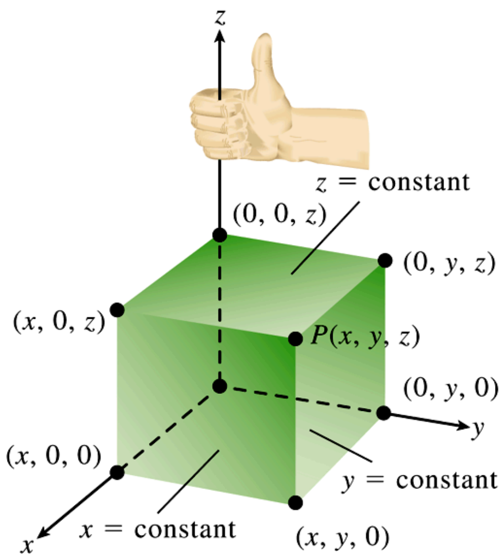
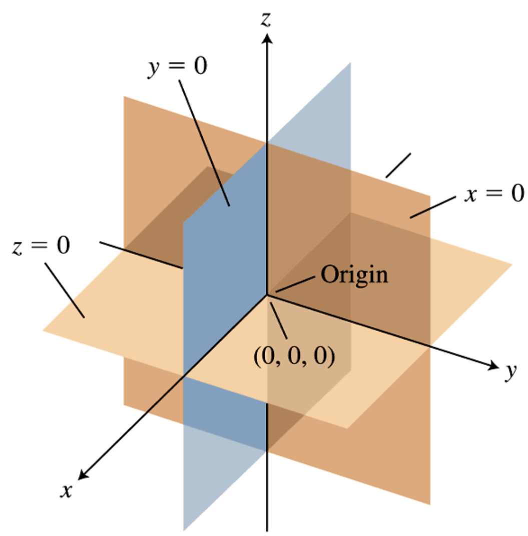
8-6 Three-Dimensional Cartesian Coordinate System

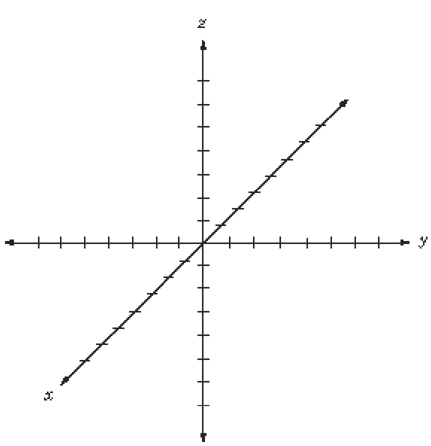
- A point is an ordered triple: (x, y, z).
- Points on the axes have the form (x, 0, 0), (0, y, 0), and (0, 0, z), where the first is on the   x-axis, the second is on the y-axis, and the third is on the z-axis.
- The coordinate planes meet at the origin (0, 0, 0).
- The coordinate planes divide space into eight regions called octants, with the first octant containing all points in space with three positive coordinates.





Example Locating a Point in the Cartesian Space

Draw a sketch that shows the points A(2, 3, 5), B(3, -4, -2), and C(-2, 5, 4)



Distance Formula (Cartesian Space)

The distance $d(P,Q)$ between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in space is

$$d(P,Q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

Midpoint Formula (Cartesian Space)

The midpoint M of the line segment PQ with endpoints $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

Example: Calculating a Distance and finding a Midpoint

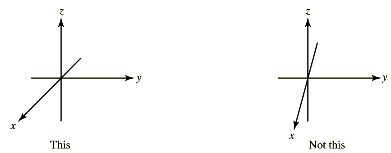
Find the distance between the points and find the midpoint of the line segment.

1. P(-2, 3, 1) and Q(4, -1, 5)
2. R(-1, 2, 5) and S(6, -3, 4)

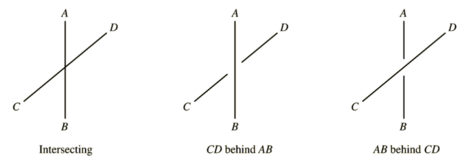
Drawing Lesson

How to Draw Three-Dimensional Objects to Look Three-Dimensional

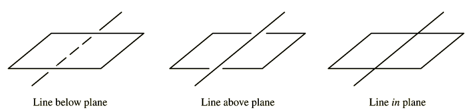
1. Make the angle between the positive x-axis and the positive y-axis large enough.



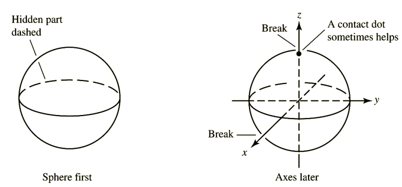
2. Break lines. When one line passes behind another, break it to show that it doesn't touch and that part of it is hidden.



3. Dash or omit hidden portions of lines. Don't let the line touch the boundary of the parallelogram that represents the plane, unless the line lies in the plane.



4. Spheres: Draw the sphere first (outline and equator); draw axes, if any, later. Use line breaks and dashed lines.



Standard Equation of a Sphere

A point $P(x, y, z)$ is on the sphere with center (h, k, l) and radius r if and only if

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2.$$

Example: Finding the Standard Equation of a Sphere.

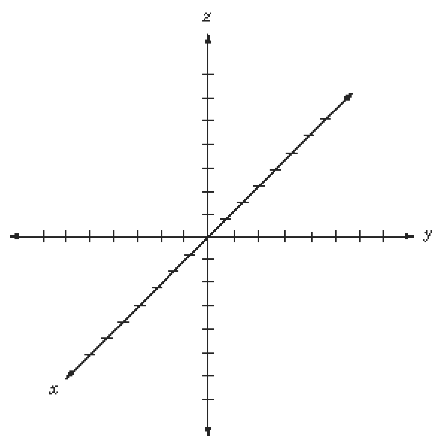
Find the standard equation of the sphere with center (1, 2, 3) and radius 4.

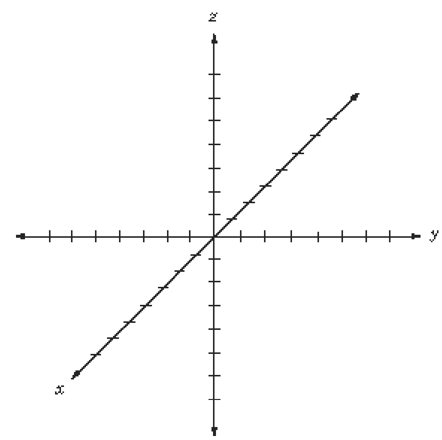
Equation for a Plane in Cartesian Space

Every plane can be written as $Ax + By + Cz + D = 0$ where A, B, and C are not all zero. Conversely, every first-degree equation in three variables represents a plane in Cartesian space.

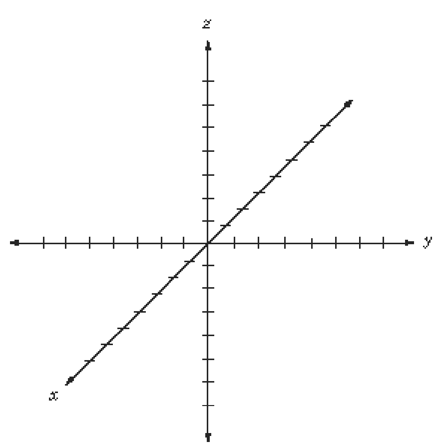
Example: Sketching a Plane in Space

Sketch the graph of 12x + 15y + 20z =60. Sketch: x + 2y + z = 6





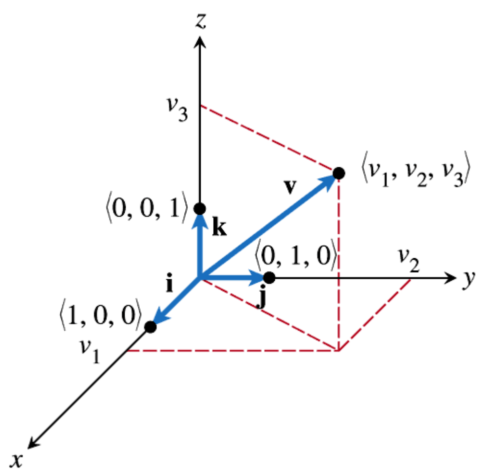
To go backwards: Write the equation for a plane with the intercepts (4,0,0), (0,6,0), (0,0,3).



Vectors in Space

In space, just as in the plane, the sets of equivalent directed line segments (or arrows) are vectors.

The Vector: **v = <**v_1, v_2, v_3**>**



The standard unit vectors are i = <1, 0, 0>,
j = <0, 1, 0>
k = <0, 0, 1>

All vector properties continue to hold.

Vector relationships in Space

For vectors $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ and $\mathbf{w} = \langle w_1, w_2, w_3\rangle$,

g**Equality:** $\mathbf{v} = \mathbf{w}$ if and only if $v_1 = w_1,\ v_2 = w_2,\ v_3 = w_3$

g**Addition:** $\mathbf{v} + \mathbf{w} = \langle v_1 + w_1,\ v_2 + w_2,\ v_3 + w_3\rangle$

g**Subtraction:** $\mathbf{v} - \mathbf{w} = \langle v_1 - w_1,\ v_2 - w_2,\ v_3 - w_3\rangle$

g**Magnitude:** $|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

g**Dot Product:** $\mathbf{v}\cdot\mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$

g**Unit Vector:** $\mathbf{u} = \mathbf{v}/|\mathbf{v}|,\ v \neq 0$, is the unit vector in the direction of **v**.

Example: Computing with Vectors:

1. 3<-2, 1, 4>
2. <0, 6, 7> + <-5, 5, 8>
3. <1, -3, 4> - <-2, -4, 5>
4. $|< 2,\ 0,\ -6 >|$
5. <5, 3, -1> • $<-6,\ 2,\ 3>$

Example: Using Vectors in Space

A jet airplane just after takeoff is pointed due east. Its air velocity vector makes an angle of 30 degrees with flat ground with the airspeed of 250 mph. If the wind is out of the southeast at 32 mph, calculate a vector that represents the plane's velocity relative to the point of takeoff.

Lines in Space

Any line in space can be written using

- one vector equation or
- A set of three parametric equations

Equations for a Line in Space

If l is a line through the point $P_0(x_0, y_0, z_o)$ and the direction of the nonzero vector **v =** <a, b, c>. Then for any point P(x, y, z) on l, $P_0P = t\cdot v$ *where v is the direction vector*.

Vector Form: r = $r_0 + tv$ *where* $r = < x, y, z >$ *and* $r_0 = < x_0, y_0, z_0 >$

Parametric Form: $x = x_0 + at,\ y = y_0 + bt,$ *and* $z = z_0 + ct$ *where t is a real number*.

Example: Finding Equations for a Line

Find the equation for a line through

$P_0(4,\ 3,\ \ -1)\ with\ direction\ vector\ v\ =\ \ <-\ 2,\ 2,\ 7\ >$ in vector form

and in parametric form.

Example: Finding Equations for a Line

Using the standard unit vectors i, j, and k, write a vector equation for the line containing the points A(3, 0, -2) and B(-1, 2, -5), and compare it to the parametric equations for the line.