7.2 Expectation Problems

Review

Ex 1 The probability distribution function of a discrete random variable X is given by

$$P(X = x) = \frac{1}{k}x^2,\ x \in \{0, 1, 2, 3, 4\}$$

a. Find k
b. Find $P(X \leq 2)$
c. Find $P(X = 3|X > 2)$
d. Find the mode of the distribution
e. Find the expected value

Ex 2 Consider the random variable Y, such that Y is obtained by increasing the values of the random variable X by a constant 3.

a. Write down an expression for Y in terms of X and 3.
b. Copy and complete the table

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | | | | | |
| P | 1/6 | 1/4 | 5/24 | 1/8 | 1/4 |

c. Find E(X)
d. Find E(Y)
e. Hence, find an expression for E(X+c), where c is constant

Ex 3 A game is developed such that a die is thrown and the banker pays out three times the value in dollars for the score on the die.

a. Construct a probability table
b. How much would you pay to make this a fair game?
c. Generalize about E(aX)
d. Generalize about E(aX+b)

Variance: a measure that describes the spread of the data. It is the square of the standard deviation. $Var(x) = \sigma^2$. $Var(x) = E\left(X^2\right) - (E(X))^2$

Ex 4 Two discs are drawn without replacement from a box containing four blue discs and three red discs. If X is discrete random variable “the number of blue discs drawn”

a. Construct a probability distribution table
b. Find E(X)
c. Find $E(X^2)$
d. Find $Var(X) = E(X^2) - (E(X))^2$
e. Find the standard deviation.

The random variable X has mean 8.1 and standard deviation 2.37. If Y=4X-7, find the mean and standard deviation of Y

- $\mathbf{E}(aX + b) = a\,\mathbf{E}(X) + b$
- $\mathbf{Var}(aX + b) = a^2\mathbf{Var}(X)$
- $\sigma(aX + b) = |a|\,\sigma(X)$

p.747#4,5
p.754#5,6
p.756#4,6