An Introduction To Reasoning

Propositional & Categorical Reasoning

The Venn Diagram Method

A star (*) indicates that there are exercises covering this section and previous unmarked sections.

This piece in relation to others: The piece is a chapter on categorical reasoning. It is independent of other chapters. The coverage here is extremely rudimentary.

Cathal Woods

The Venn Diagram Method

1 Categorical Generalizations

1. Consider the following propositions:

(a) All males are humans.
(b) Most physicists are male.
(c) Few teachers are rock-climbers.
(d) No dogs are cats.
(e) Some Americans are doctors.
(f) Some adults are not logicians.

(a) – (f) are *categorical generalizations*. That is, they are about categories or classes or types of things. (a), for example, is about the class of males and the class of humans. They make no mention of any particular members of the categories or classes or types they are about.

2. The propositions are also quantified in that they state what proportion of a class does or does not belong to another class. Some categorical generalizations are universal in quantity while some are partial. For instance, (a) and (d) are universal: (a) quantifies over the entire class of males, saying that all of its members are in the class of humans; (d) quantifies over the whole class of dogs, saying that none of its members is in the class of cats. In contrast, (b), (c), (e) and (f) are partial: (b) says that most but not all physicists are males; (c) says that some but not many teachers are rock-climbers; (e) says that says that some Americans are in the class of doctors and (f) says that some adults are not in the class of logicians. The word "Some" in (e) and (f) does not tell us very accurately what the proportion is, but at a minimum, it tells us that there is at least one member of the first class which is also a member of the second. Thus (e) states that, of the class of Americans, at least one is also in the class of doctors.

3. Some generalizations are positive, and some are negative. (a), (b) (c) and (e) are positive: (a), for example, says what is in the class of humans, as opposed to saying what is not in the class of humans, and (e), likewise, says what is a doctor, in contrast to saying what is not a doctor. (d) and (f), on the other hand, are negative: (b) says what is not in the class of cats, and (d) says what is not a logician.

4. Using the Venn diagram method, we can evaluate inferences which involve propositions of four of the forms above: positive universals, such as (a), which have the form "All *S*s are *P*s."; negative universals, such as (d), which have the form "No *S*s are *P*s."; positive partials, such as (e), which have the form "Some *S*s are *P*s."; and negative partials, such as (f), which have the form "Some *S*s are not *P*s.". (Capital letters in italics stand for any class.)

Propositions such as (b) and (c) which use quantifiers other than "All", "No", and "Some" are *not* used in the Venn diagram method, since they do not yield inferences which can reliably be classified as valid or invalid when combined with each other or with categorical generalizations of other forms (though they can be combined with propositions about particulars in an instantiation syllogism — see *I&S – Induction, section 3*). For example, although the two propositions "Many teachers are rock-climbers." and "Many rock-climbers are people who lift weights." can be connected by the term "rock-climbers", it is not clear that we could safely conclude that many teachers lift weights, though we could conclude, with reasonably high confidence, that *some* teachers lift weights. The combination of "Many assistance animals are dogs" and "All dogs are four-legged things." reliably yields "Many assistance animals are four-legged things.", but, in general, propositions with indeterminate quantification, such as "many", "few", "a majority of", "a lot of", and so on, cannot usefully be described in general (that is, in a method) and must be examined on a case-by-case basis.

From now on, then, we shall deal only with universal and partial categorical generalizations.

2 Some Extras On Categorical Generalizations

1. For the purposes of the Venn diagram method, the propositions involved must follow a specific form, beginning with "All", "Some" or "No", followed by the first (subject) category, followed by "are" or "are not", and finally the second (predicate) category. In everyday life, however, lots of categorical generalizations are disguised.

2. The sentence "Some roses are red." should be translated as "Some roses are red flowers.", or "Some roses are red things.". Likewise, "All professional football players are strong." should be "All professional football players are strong persons.", or "All professional football players are strong things.".

The lesson is that for the Venn diagram method the subject and predicate terms need to be plural nouns, such as "cats", "paper-clips", "networks". In short, they need to be names of a class of things, as opposed to names of properties that things have. If you are in any doubt about the kind of group formed by a property, any property can be turned into a plural noun by adding "thing". For example, "Giraffes are tall." can be rewritten as "Giraffes are tall things.".

3. The sentence "No cats bark." should be translated as "No cats are animals that bark.", or "No cats are things that bark.". In the same way, "All birds can fly." should be "All birds are animals that can fly.", or "All birds are things that can fly.".

The lesson is that for the Venn diagram method the verb needs to be either "are" or "are not".

4. Sometimes categorical generalizations come without an explicit quantifier:

(a) Males are humans.
(b) A male is a human.
(c) Dogs are not cats.
(d) A dog is not a cat.

(a) and (b) should be translated as "All males are humans.", and (c) and (d) as "No dogs are cats.".

5. The quantifiers "all" and "some" can be disguised:

(a) Every male is human.
(b) Whatever is male is human.
(c) Only humans are males.
(d) None but humans are males.
(e) Whatever is a dog is not a cat.
(f) Not a single dog is a cat.
(g) There are Americans that are doctors.
(h) Someone in America is a doctor.
(i) At least a few Americans are doctors.
(j) Not everyone who is an adult is a logician.

For use in the Venn diagram method, (a)-(d) should be translated as "All males are humans."; (e) and (f) as "No dogs are cats."; (g)-(i) as "Some Americans are doctors.". and (j) as "Some adults are not logicians.".

6. Some conditionals can be expressed as categorical generalizations:

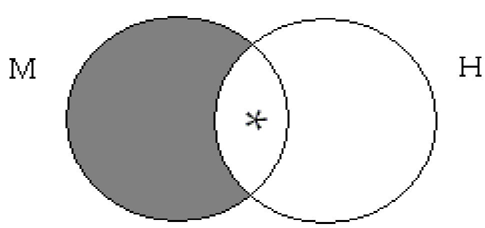
(a) If someone is a male, then he is a human.
(b) A thing is a human if it's a male.
(c) If something is not a human, it's not a male.
(d) If something is a dog, then it's not a cat.
(e) If something is a cat, then it's not a dog.
(f) Something is a dog only if it's not a cat.

(a), (b), and (c) should be translated as "All males are humans." and (d), (e), and (f) as "No dogs are cats.".

7. Sentences about specific individual things can be expressed as categorical generalizations. The sentence "Paul is tall." comes to "All persons identical to Paul are tall persons.", or "All persons identical to Paul are tall things.".

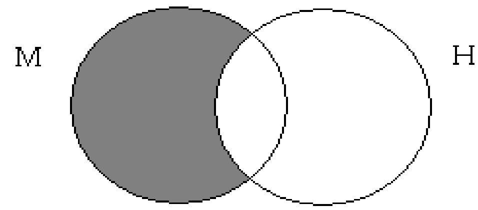
3 Venn Diagrams & Categorical Generalizations

1. Consider the following diagram:



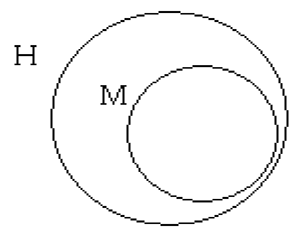
This is an example of a Venn diagram. The circle on the left represents the class of males (M), the circle on the right represents the class of humans (H). The shading of the non-overlapping part of M is a 'shading out'; it indicates that nothing is both a male and a non-human, or, that all males are humans. The asterisk in the overlap indicates that at least one thing is both a male and a human. So in total, the diagram says that nothing is both a male and a non-human, and that something is both a male and a human.

2. (a) from above — a positive universal — gets represented like this in a Venn diagram:



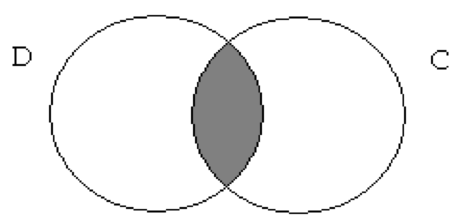
The claim, remember, is that if something is a male then it is a human, or, in other words, that every male is human, or, that nothing is both a male and a non-human. Thus the part of the Males-circle not overlapping the Humans-circle is shaded, representing the fact that that part of the Males-circle is empty.

Positive universals could also be diagrammed like this:



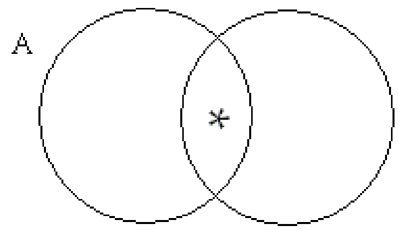
The Males-circle is wholly inside the Humans-circle, thus representing the fact that all of the members of the class of males are members of the class of humans. In our system, however, we use two (or more) overlapping circles, rather than circles within circles, so that propositions of different forms can be represented on the same basic diagram.

3. (d) — a negative universal — gets represented thus:



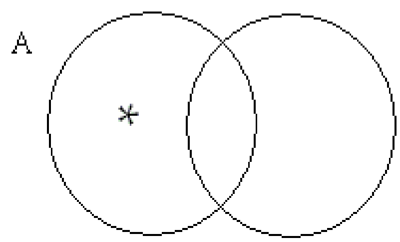
The claim is that if something is a dog then it is not a cat, or, put another way, that nothing is both a dog and a cat. Thus the part of the Dogs-circle overlapping the Cats-circle is shaded, representing the fact that that part of the Dogs-circle is empty.

4. (e) — a positive partial — gets represented like this:



The claim is that at least one American is a doctor, that something is both an American and a doctor. Thus the part of the Americans-circle overlapping the Doctors-circle has an asterisk in it, representing the fact that that part of the Americans-circle is not empty.

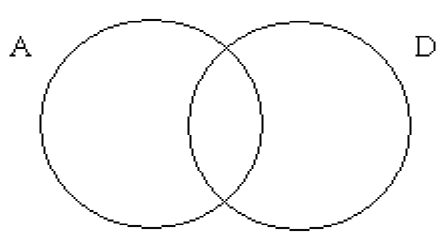
5. (f) — a negative partial — gets diagrammed as follows:



The claim is that at least one adult is not a logician, that something is both an adult and a non-logician. Thus the part of the Humans-circle not overlapping the Logicians-circle has an asterisk in it, representing the fact that that part of the Adults-circle is not empty.

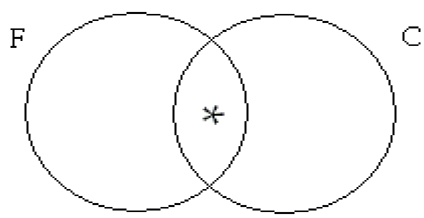
4 Existential Commitment

1. *If* we were to assume that there is always at least one member to every class, positive and negative partials could together be diagrammed thus:



If "A" is Americans and "D" is doctors, the overlapping part of the Americans-circle (A) overlaps the Doctors-circle (D) would represent the fact that some of the members of the class of Americans are members of the class of doctors; and part of the Americans-circle does not overlap the Doctors-circle, thus representing the fact some of the members of the class of Americans are not members of the class of doctors, and similarly for doctors who are not Americans.

In our system, however, the mere fact that an area is open tells us nothing and an asterisk is required to indicate the existence of a member of a class. Consider the following diagram:



The Students-Failing-The-Final-circle (F) and the Students-Failing-The-Class-circle (C) overlap, but in order to represent the fact that there is at least one student who fails an asterisk in the overlap is required.

In other words, our system assumes that classes are empty unless an asterisk explicitly states otherwise, whereas the alternative operates on the assumption that there are members in every area of the diagram which is not shaded out. The issue here is *existential commitment*, that is, whether categorical propositions, and universal propositions in particular, imply that there are entities of which the proposition is true, or, alternatively, whether the proposition is true even though nothing satisfies it

2. Consider (e) and (f) from the list of propositions in section 1 above. (e) is the proposition "Some Americans are doctors." and (f) is the proposition "Some adults are not logicians.". It is clear that in asserting (e), one is committed to the existence of Americans, in that one is saying in part that the class of Americans has at least one member. Moreover, it is equally clear that in asserting (f), one is committed to the existence of adults: one is saying in part that the class of adults is not empty—that it has at least one member. (e) and (f), thus, involve existential commitment, and propositions involving "some" can thus be called "existential" in addition to "partial". This is true generally: all positive existentials and all negative existentials involve existential commitment. In saying that some *S*s are *P*s one is committed to the existence of *S*s, and the same for saying that some *S*s are not *P*s.

3. But what about positive universals and negative universals? In saying that all *S*s are *P*s, is one saying in part that the class of *S*s has at least one member? In saying that no *S*s are *P*s, is one saying that *S*s exist? Consider the propositions "Any student who fails the final fails the class." and "All breakages must be paid for." (which we might rewrite as "All persons breaking items are persons who pay for those items."). Intuitively, such propositions do *not* have existential commitment.

The worry arises because of inferences such as "All roses have thorns. So, some thorny things are roses.". This inference can land us in trouble if the class is empty, as for example when we move from "All people contacting SARS will be quarantined." to "Some people who are quarantined are people who have SARS." The universal proposition remains true even if no one contacts SARS, which would make the existential proposition false, since it has existential commitment. The inference "All *S*s are *P*s. So, some *P*s are *S*s." might thus be invalid, in that it is possible to move from a true premise to a false conclusion. The same is true for the inference "All *M*s are *P*s. All *M*s are *S*s. So, some *S*s are *P*s." (e.g. "Anyone who breaks something must pay for it. Anyone who breaks something will be asked to leave. So, some people who will be asked to leave are people who pay for what they broke.").

Obviously, there are two options. The first is to say that positive and negative universals involve existential commitment. The sentence "All males are humans." comes to "The class of males has members, and each such member is a human.", and "No dogs are cats." comes to "The class of dogs has members, and no such member is a cat.". The second option is to say that universals carry no such commitment: "All males are humans." comes to "If something is a male, then it is a human.", and "No dogs are cats." comes to "If something is a dog, then it is not a cat.". So, whereas on the first option universals are conjunctions involving the claim that the class of *S*s has members, on the second option they are conditionals involving no such claim.

The modern logician opts for the second option. She argues, first, that everyday linguistic practice is inconclusive on whether we should go with the first option or the second. As we have just seen, usually when we give positive and negative universals we take there to be *S*s and, thus, the fact that a person gives a positive or negative universal makes it highly likely that he takes there to be *S*s, but sometimes when we give positive and negative universals we are non-committal on the existence of *S*s. When a teacher tells his students on the first day of classes that all students failing the final will fail the class, he is not thereby committed either way to the existence of students who will fail the final. The modern logician argues, second, that treating universals as conditionals involving no existential commitment makes logic simpler and more powerful.

The lesson, thus, is that whereas asserting a positive or negative particular involves asserting the existence of *S*s, this is not the case with asserting a positive or negative universal. The proposition "All students who fail the final are students who fail the class." does not commit itself to the existence of students-who-fail-the-final.

5 Immediate Inferences, Categorical Syllogisms & The Venn Diagram Method*

1. Consider the following inferences:

1. No dogs are cats.

----

2. No cats are dogs.

1. All males are humans.
2. All humans are animals.

----

3. All males are animals.

The first inference is an *immediate inference*, for it is made up exclusively of categorical generalizations, and it has exactly one premise. The second inference, in contrast, is a *categorical syllogism*: it is made up exclusively of categorical generalizations, just like the first inference, but it has exactly two premises, as opposed to just one.

2. For immediate inferences, the *Venn diagram method* works like this. The first step is to make a translation key for the *S*-term and the *P*-term, where the *S*-term is the subject term *in the conclusion* and the *P*-term is the predicate term *in the conclusion*:

C: cats
D: dogs

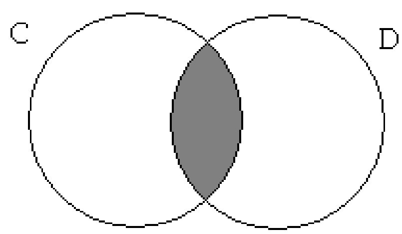
The second step is to put the inference in standard form relative to the key:

1. No Ds are Cs.

----

2. No Cs are Ds.

The third step is to diagram the premise. We put the circle for the subject of the conclusion on the left and the circle for the predicate on the right:



The part of the D-circle overlapping the C-circle is shaded, thus representing the fact that that part of the D-circle is empty: no Ds are Cs. The fourth, and final, step is to determine whether the diagram now shows us that the conclusion ("No Cs are Ds.") must be true. If it were, the part of the C-circle overlapping the D-circle would be shaded. Given that that part of the C-circle is in fact already shaded, the inference is valid.

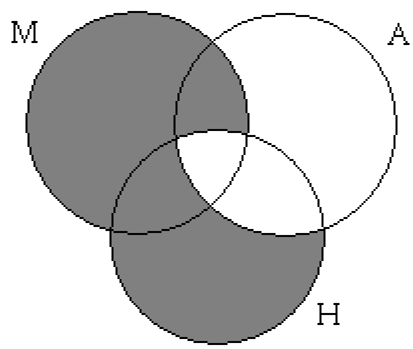
3. For categorical syllogisms, the method works just like it does for immediate inferences except that there are three terms and three circles instead of just two. We construct the diagram as follows: we look at the conclusion and put the subject term of the conclusion first, then draw an overlapping circle to the right for the predicate term of the conclusion and we draw the third below and in the middle of those two and label it with the other term which appears in the inference. The translation key for the inference above about males, thus, looks like this:

M: males
A: animals
H: humans

In standard form and relative to the key, it looks like this:

1. All Ms are Hs.
2. All Hs are As.
   ----
3. All Ms are As.

The information in the premises gets diagrammed thus:



The shading in the part of the M-circle not overlapping the H circle represents the information in the first premise, and the shading in the part of the H-circle not overlapping the A-circle represents the information in the second premise. The conclusion is necessitated by the diagram: the part of the M-circle not overlapping the A-circle is already shaded; it must be the case that all Ms are As. So, the inference is valid. Note that if the left part of the overlap between the M-circle and the H-circle were not shaded in, the inference would not be valid, because it would be possible for there to be some Ms which are not As.

Here is another example:

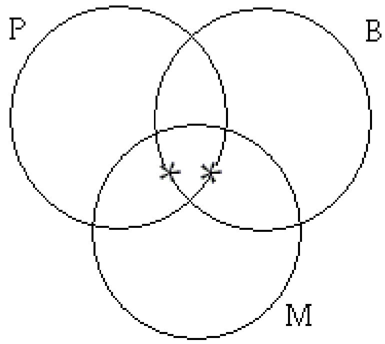
> Some philosophers are males, and some males are billionaires Some philosophers, therefore, are billionaires.

With "P" standing for "philosophers", "B" standing for "billionaires", and "M" standing for "males", the inference looks like this in standard form:

1. Some Ps are Ms.

2. Some Ms are Bs.
   ----
3. Some Ps are Bs.

The information in the premises gets diagrammed thus:



Representing the information in the first premise requires putting an asterisk in the part of the P-circle overlapping the M-circle. But since that part of the P-circle itself has two parts, representing the information in the first premise requires putting an asterisk on the line separating the two parts. (You might want to make the asterisk larger than shown here, so that it clearly overlaps both subsections of the intersection.) The same goes for representing the information in the second premise, thus the asterisk on the line in the part of the M-circle overlapping the B-circle. And notice, the conclusion is not necessitated by the diagram. For if it were, then there would be an asterisk in the part of the P-circle overlapping the P-circle, but since the two asterisks in the diagram could be in the parts of the P and B circles which do not overlap, we cannot be sure that there is asterisk in the overlap between the P-circle and the B-circle. The conclusion is not necessarily true, and thus, the inference is not valid.

The Venn Diagram Method

Exercise Set (1) | Exercise Set (2)