INTEGERS

INTEGERS

A) Understanding Integers

1. An integer is a whole number with the positive

sign or negative sign, including zero which is

neither positive nor negative.

Integers
- Negative integers
  For example:-
  -32, -8, -3
- 0 (zero)
- Positive integers
  For example:-
  +1, +10, +29
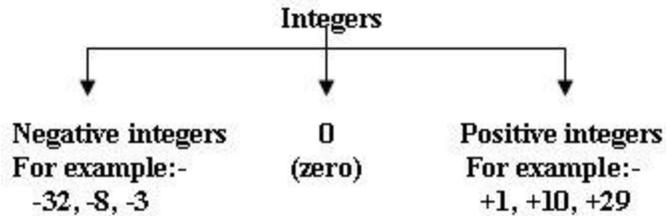
2. Positive and negative numbers include decimals

and fractions.

For example :-

(a) Positive numbers : +12, 8.5, + 1$\frac{1}{2}$

(b) Negetive numbers : -15, - 3.6, - 3$\frac{3}{4}$

3. Positive integers can be written without the '+' sign.

For example :-

+4 is usually written as 4.

<u>**Worked Example 1**</u>

$$+8, \; 4\frac{1}{2}, \; -5, \; 0, \; 6.3, \; -2, \; -4.5, \; 18, \; -2\frac{1}{2}$$

From the numbers above, list all the

(a) integers,

(b) positive integers,

(c) negative integers.

<u>**Solution**</u>

(a) - 5, -2, 0, +8 and 18

(b) +8 and 18

(c) -5 and -2

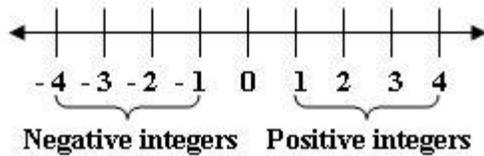## B) Representing Integers on Number Lines

1. Integers can be represented on either a horizontal

   or a vertical number line.

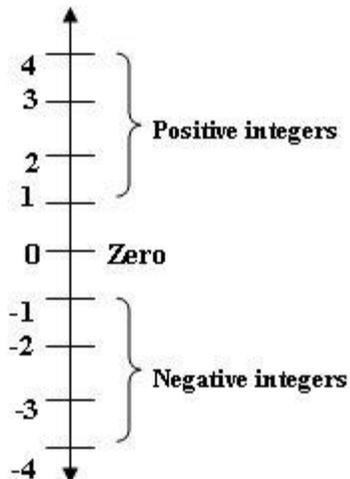2. Normally, a horizontal number line is used.

   For example :-

(a) Horizontal number line
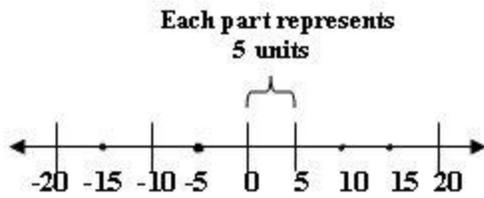

(c) Vertical number line

Worked Example 2

Draw a number line and mark the following numbers :-

- 15 , - 5, 10, 15

<u>Solution</u>

Each part represents
5 units

-20 -15 -10 -5 0 5 10 15 20

C) Comparing the Value of Two Integers

We can use the number line to compare the values

of integers. A given integer, x, is greater than all the

integers on its left but smalller than all the integers

on its right.

Integers
smaller than x   Integers
                   greater than x

x
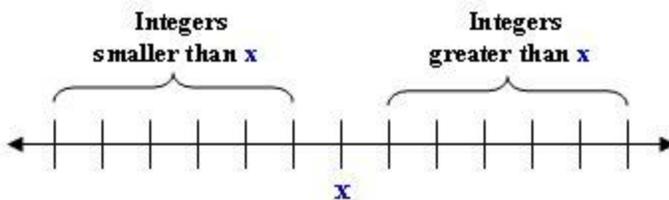
Worked Example 3

In each of the following pairs of integers, state the

integer that is greater in value.

(a) - 3, 2   (b) - 6, - 3

Solution

(a)

-3 -2 -1 0 1 2

2 is on the right of -3.
Therefore, 2 is greater than -3.

(b)

-7 -6 -5 -4 -3 -2

- 3 is in the right of –6.
Therefore, -3 is greater than -6.

Worked Example 4

State the integer with the smaller value : - 5 or 0
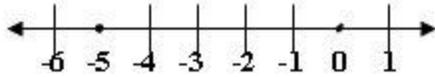
-5 is on the left of 0.

Therefore, -5 is less than 0.

D) Arranging Integers in Order
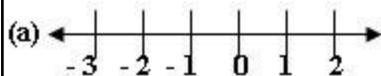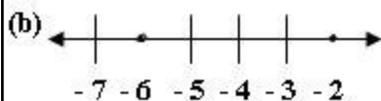
1. In increasing order, the integers are arranged

   from the smallest to the largest.

   For example :-

   - 8, -5, -1, 0, 3, 7, 10
   —————————————→
   Value increasing
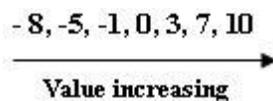
2. In decreasing order, the integers are arranged

   from the largest to the smallest.

   For example :-

   10, 8, 6, 4, 0, -4, -6, -9
   —————————————→
   Value decreasing
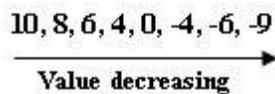
Worked Example 5

(a) Arrange the integers -6, 2, -1, +4, 0, - 2 in

   increasing order.

(b) Arrange the integers 3, 0, 5, - 5, 6, - 4 in

   decreasing order.

Solution

(a) - 6, - 2, -1, 0, 2, +4

(b) 6, 5, 3, 0, - 4, - 5

Worked Example 6

Determine the largest integer and the smallest

integer in the following : 8, - 3, - 14, - 9, - 4, 7

Solution
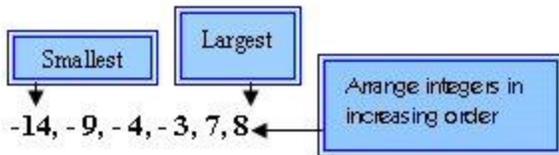
Therefore, 8 is largest integer and – 14 is

the smallest integer.

F) Completing a Sequence of Integers

To complete a sequence of integers, it is necessary

to identify the pattern of the sequence.

Worked Example 7

- 10 *P* - 6 - 4   *Q*   *R*
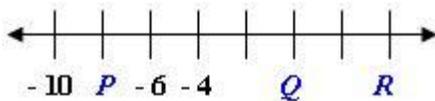
State the integers that are represented by the letters

_P_, _Q_ and _R_ on the number line above.

Solution

    P   *Q*   *R*

-10 -8   -6 -4 -2   0   2   4

+2 +2 +2   +2 +2 +2 +2
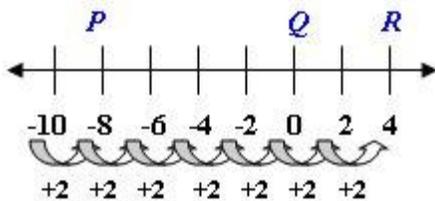
Therefore, *P* = -8, *Q* = 0, *R*= 4

Worked Example 8

Complete the following sequence of integers.

- 12, - 9, - 6, __, __, __

Solution
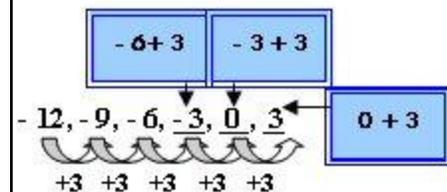
**G) Using Positive and Negative Integers
in Real- life Situations**

Positive and negative integers can be used in

the following contexts in real-life situations.

(a) An increase or a decrease in value

   For example :-

   (i) +RM100 represents an increase of Rm100

      in price whereas -RM50 represents a decrease

      of RM50 in price.

   (ii) +60 sen represents a profit of 60 sen whereas

      - 30 sen represents a loss of 30 sen.

   (iii) +4 kg represents a mass gain of 4 kg whereas

      - 6 kg represents a mass loss of 6 kg.

(b) A value which is either greater than or less than

   zero.

   For example :-

(i) +5°C represents 5⁰C more than 0⁰C whereas

- 2⁰C represents 2⁰C less than 0⁰C.

(ii) As 0 m is the sea level, +3 m represents 3 m

above sea level whereas -10 m below sea

level.

(c) A positive or a negative direction (i.e.opposite

directions)

For example :-

(i) If +8 km represents 8 km to the east, then -7 km

represents 7 km to the west.

(ii) If a lift which moves 4 floors upwards is repre-

sented by +4, then a lift which moves downwards

is represented by - 3.

Worked Example 9

Use positive or negative integers to represent the

following.

(a) A temperature rise of 18^0C

(b) 800 m below sea level

(c) A loss of RM70
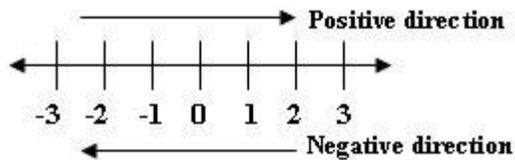
Solution

(a) $+18^0$C   (c) -RM70

(b) - 800 m

ADDITION AND SUBTRACTION
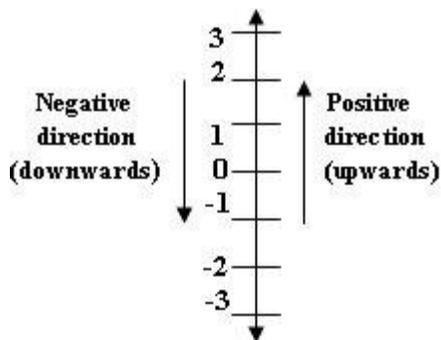## OF INTEGERS

A) Addition of integers

1. **For a horizontal number line, the direction from left to right indicates the positive direction while the direction from right to left indicates the negative direction.**

   For example :-

2. **For a vertical number line, and upward direction indicates the positive direction indicates the negative direction.**

   For example :-

3. **We can add two or more integers using a number line.**

   (a) When adding a positive integer, we move in the positive direction on the number line.

   (b) When adding a negative integer, we move in the negative direction on the number line.

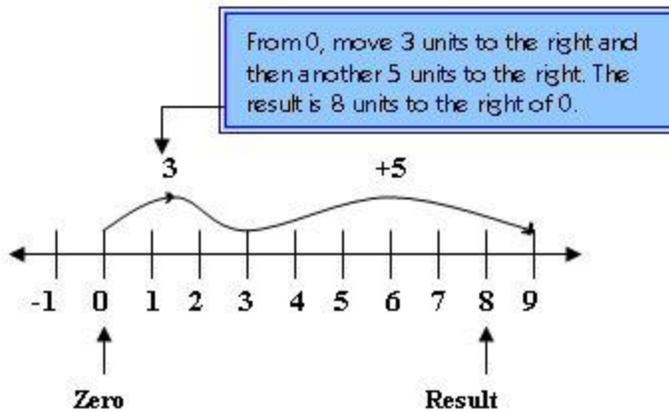   **(c) Zero is usually taken as the starting point.**

Worked Example 10

Solve the following by using the number lines.

(a) 3 + (+5)   (c) - 2+ (- 5)

(b) - 6 + 4   (d) 3 + (- 5)+(+8)

Solution

(a)

From 0, move 3 units to the right and then another 5 units to the right. The result is 8 units to the right of 0.

3   +5

-1   0   1   2   3   4   5   6   7   8   9

Zero   Result

Therefore, - 6 + 4 = - 2

(b)

From 0, move 6 units to the left and then 4 units to the right. The result is 2 units to the left of 0.

-6

+4

-7 -6 -5 -4 -3 -2 -1 0   1
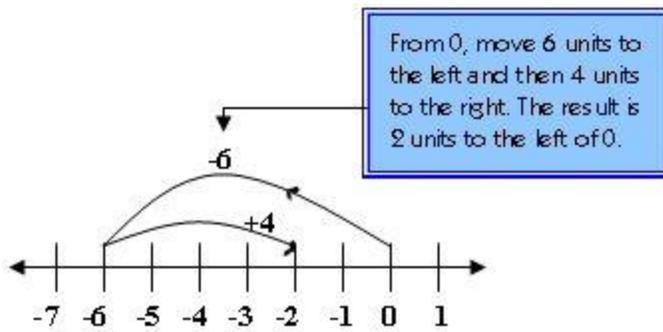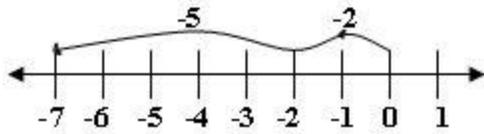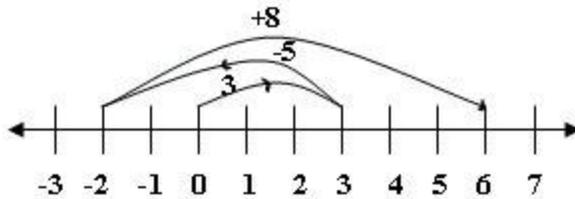
Therefore, - 6 + 4 = - 2

(c)

Therefore, -2 + (-5) = -7

(d)

Therefore, 3 + (-5) + (+8) = -2 + (+8)

$$= +6$$

$$= 6$$

Worked Example 11

Solve each of the following.

(a) - 9 + (+5)   (c) - 6 + (- 2)

(b) + 7 + (- 5)

Solution

(a) $-9 + (+5) = -9 + 5$

$$= -4$$

(b) $+7 + (-5) = +7 - 5$

$$= +2$$

$$= 2$$

(c) $-6 + (-2) = -6 - 2$

$$= -8$$

B) Problem Solving involving Addition of Integers

Worked Example 12

A submarine which is 9 km below sea level

moves up 3 km. What is the position of the

submarine now ?

Solution

1. **Understand the problem**

   Given information :

   The submarine is 9 km below sea level.

   It moves up 3 km.

   Find : Final position of the submarine
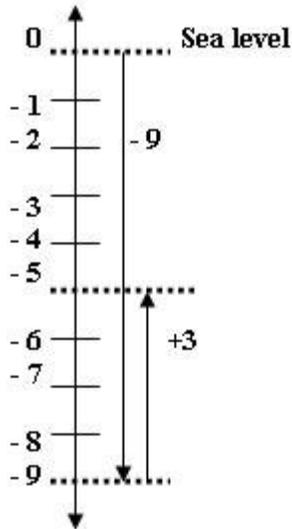
2. **Devise a plan**

   Use addition.

3. **Carry out the plan**

   - 9 + 4 = - 5

   Therefore, the submarine is 5 km below sea

   level now.

4. **Check**

C) Subtraction of Integers

1. Subtraction of integers is to find the difference

   between two integers.

2. We can use a number line to find the difference

   between two integers.

3. The difference can be determined by counting the

   number of units to be moved from the second integer

   to the first integer. The direction of the movement will

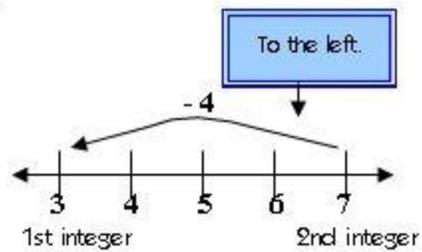   determine the '+' or '_' sign in the answer.

Worked Example 13

Solve the following by using number line.

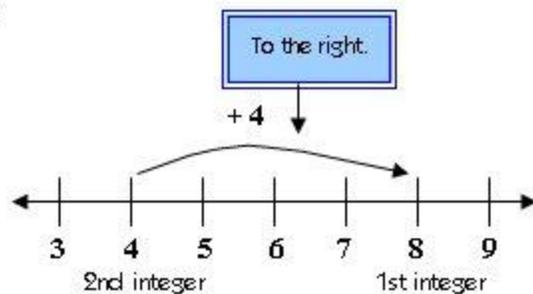(a) 3 - 5   (c) - 2 - 4

(b) 8 - (+4)   (d) 0 - (- 7)

Solution
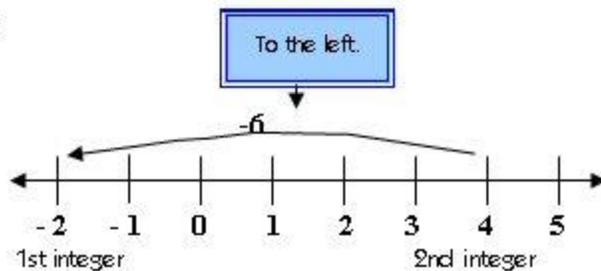
(a)

To the left.

- 4
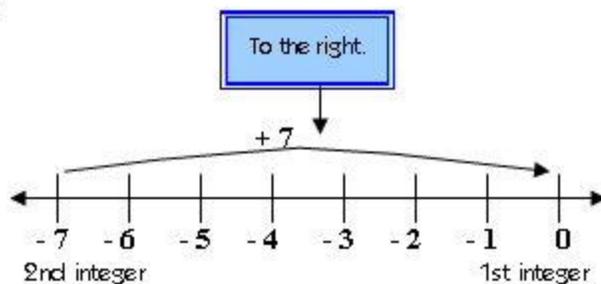
```
←———|———|———|———|———|———→
    3   4   5   6   7
  1st integer        2nd integer
```

Therefore, $3 - 5 = -4$

(b)

To the right.

+ 4

```
←——|———|———|———|———|———|———|——→
   3   4   5   6   7   8   9
     2nd integer        1st integer
```

Therefore, $8 - (+4) = +4 = 4$

(c)

To the left.

-6

```
←——|———|———|———|———|———|———|———|——→
  -2  -1   0   1   2   3   4   5
 1st integer              2nd integer
```

Therefore, $-2 - 4 = -6$

(d)

To the right.

+ 7

```
←——|———|———|———|———|———|———|———|——→
  -7  -6  -5  -4  -3  -2  -1   0
 2nd integer                1st integer
```

Therefore, $0 - (-7) = +7 = 7$

Worked Example 14
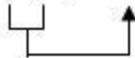
Find the value of each of the following.
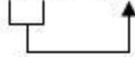
(a) 8 - (+5)

(b) 6 - (- 3)
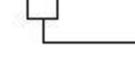
(c) - 9 - (- 9)

Solution

(a) $8 - (+5) = 8 - 5 = 3$

(b) $6 - (- 3) = 6 + 3 = 9$

(c) $-9 - (- 9) = -9 + 9$

$= 0$

Worked Example 15

Find the value of each of the following.

(a) 30 - (+18) - (- 6)

(b) - 21 - (- 7) - (- 13)

Solution

(a) 30 - (+18) - (- 6) = 30 - 18 + 6

$= 12 + 6$

$= 18$

(b) - 21 - (- 7) - (- 13) = - 21 + 7 + 13

$= - 14 + 13$

$= -1$

D) Problem Solving involving Subtraction of Integers

A submarine is 8 m below sea level. An eagle is 3 m

above sea level directly above the submarine. What

is the distance between the submarine and the eagle?

Solution

1. Understand the problem

Given information :

A submarine is 8 m below sea level.

An eagle is 3 m above sea level.

Find : Distance between the submarine

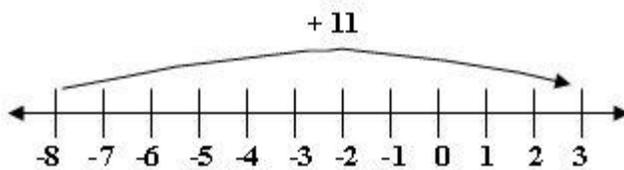and the eagle

2. Devise a plan

Use subtraction.

3. Carry out the plan

3 - (- 8) = 3 + 8 = 11

Therefore, the distance between the submarine

and the eagle is 11 m.

4. Check

E) Combined Operations of Addition and Subtraction of Integers

Worked Example 17

Find the value of each of the following.

(a) 25 - (+30) + (- 12)

(b) - 12 - (- 26) + (- 15)

Solution

(a) 25 - (+30) + (- 12)

   = 25 - 30 - 12

   = - 5 - 12

   = - 17

(b) - 12 - (- 26) + (- 15)

   = - 12 + 26 - 15

   = 14 - 15

   = - 1

F) Problem Solving involving Addition and Subtraction of Integers

Worked Example 18

A diver at 7 m below sea level swam up 3 m. A

turtle at that moment was 2 m below sea level.

What was the distance between them when the

turtle was directly above the diver?

Solution

1. Understand the problem

   Given information :

   The diver was 7 m below sea level at first.

   He swam up 3 m.

   The turtle is 2 m below sea level.

   Find : Distance between the diver and the turtle.

2. Devise a plan

Use addition and subtraction.

3. **Carry out the plan**
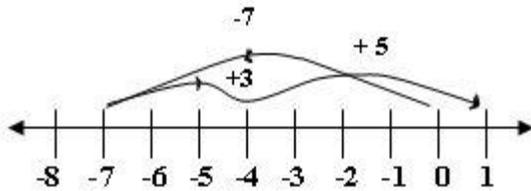
   (- 7 + 3) - (-5) = (- 7 + 3) + 5

   $\qquad$ = - 4 + 5

   $\qquad$ = 1

   Therefore, the distance between the diver and

   the turtle was 1 m.

4. **Check**