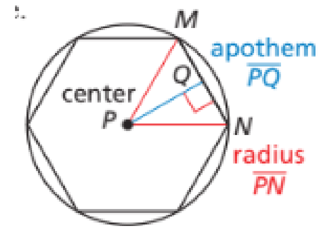
Geometry	Lesson 9.6: The Central Angle and Area of Regular Polygons	Notes

CENTRAL ANGLE OF A POLYGON NOTES

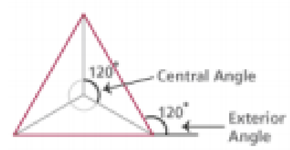
- A polygon <u>drawn inside of a circle so that its vertices touch the circle</u> is said to be an ***inscribed polygon***.
  - o The circle itself is referred to as a *circumscribed circle* due to being outside the polygon.
- In looking at the *inscribed polygon* in the circle, its helps to give names to special line segments and other angles of the polygon, such as:
  - i. The *center* of a polygon is the point also at the center of the *circumscribed circle*.
  - ii. The *radius* of a polygon is a segment connecting the center and any vertex of the polygon.
  - iii. The ***apothem*** (ă-pa-them) is the perpendicular bisecting segment connecting the center of the polygon with the mid-point of any side of the polygon.
    - ■ It is also the height of each triangle created by joining the center of the polygon with two consecutive vertices of the polygon.
  - iv. The angle between two consecutive radii of the polygon is referred to as the polygon's ***central angle***.

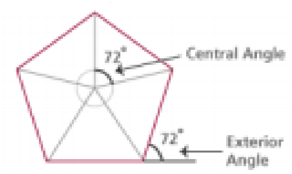


∠MPN is a central angle.

★ To find the measure of the <u>central angle of a **regular** polygon</u>, divide $360°$ by the number of sides, n.

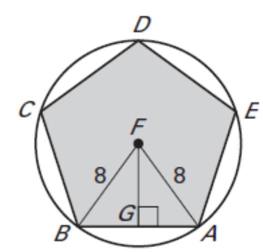
$$Measure\ of\ \ Central\ Angle\ =\ \frac{360°}{n}$$





EXAMPLE 1 – Use the figure of regular polygon $ABCDE$ to answer the following questions.

1. Identify the *apothem of the polygon*.	2. Identify a *radius of the polygon*.
3. Find $m\angle AFB$.	4. Find $m\angle AFG$.



AREA OF A POLYGON NOTES

<u>Area of a Regular Polygon</u>

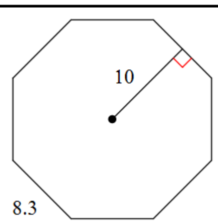
- To find the area of a *regular polygon*, use
  - o a is the <u>apothem length</u>
  - o n is the <u>number of sides</u> of the polygon
  - o s is the <u>side length</u> of the polygon

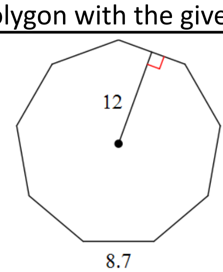
$$A = \frac{1}{2}ans$$

- To find the perimeter, multiply the number of sides *(n)* by the length of each side *(s)*. $P = ns$
  - o Combine the two formulas and find the area when given the perimeter by $A = \frac{1}{2}aP$

EXAMPLE 2 – Find the area of the regular polygon with the given information.

1.	2.	3. A decagon with a = 17 and s = 11.

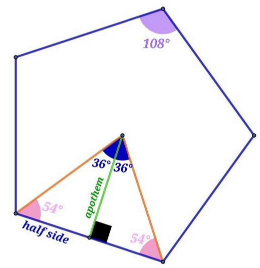




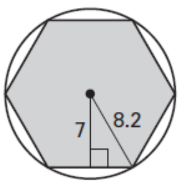
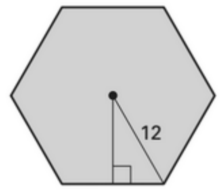
FINDING APOTHEM AND SIDE LENGTHS

- Remember, the *apothem* is perpendicular to the side of a polygon.
  - ★ So we will ***most likely*** use trig *(SOH-CAH-TOA)* to find
    - i. *half the side length (s)* of the polygon
    - ii. length of the *apothem (a)*
- If you are going to use trig, then you **must** use *half the measure of the central angle*.

☆ You can also use Pythagorean Theorem ($a^2 + b^2 = c^2$) *if possible.*



EXAMPLE 3 – Find the length of the side **and** apothem of the regular polygon.

1.	2.
	

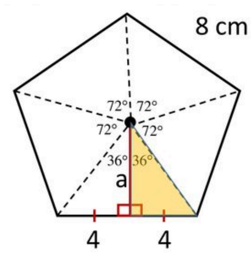
FINDING AREA GIVEN ONLY THE APOTHEM OR A SIDE LENGTH

Finding the Area with Only a Known Side Length

I. Create a small *right triangle* using:
   - i. The *apothem.*
   - ii. *Half* of the *central angle*.
   - iii. *Half* of the **given** *side length*.

$$\tan\left(\frac{Central\ Angle}{2}\right) = \frac{\left(\frac{Side\ Length}{2}\right)}{Apothem}$$

II. And then use ***trigonometry*** to solve for the unknown *apothem*.



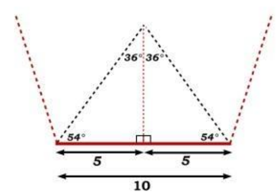
Finding the Area with Only a Known Apothem

I. Create a small *right triangle* again:
   - i. The *apothem*.
   - ii. ***Half*** of the *central angle*.
   - iii. ***Half*** of the unknown side length

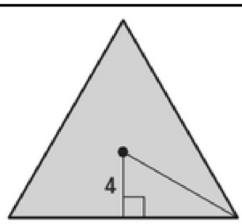
$$\tan\left(\frac{Central\ Angle}{2}\right) = \frac{Apothem}{\left(\frac{Side\ Length}{2}\right)}$$

II. And then use *trigonometry* to solve for **half** the *unknown* side length.

★ ***Don't forget to DOUBLE the answer to create the side length before finding the area of the polygon.***



EXAMPLE 4 – Find the area of the regular polygon with the indicated given conditions.

Only a Known Apothem	Only a Known Side Length
1. 	2.