



## Flipping Physics Lecture Notes:

Velocity as a function of Position in Simple Harmonic Motion  
<http://www.flippingphysics.com/shm-velocity-position.html>

We have already derived<sup>1</sup> this equation for total mechanical energy of a mass-spring system:

$$ME_t = \frac{1}{2}kA^2$$

At any point in the motion of the mass-spring system, it will have both kinetic and elastic potential energies:

$$ME_t = KE + U_e = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

And we can rearrange that equation to solve for velocity as a function of position:

$$\Rightarrow mv^2 = kA^2 - kx^2 = k(A^2 - x^2) \Rightarrow v^2 = \frac{k}{m}(A^2 - x^2) \Rightarrow v = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

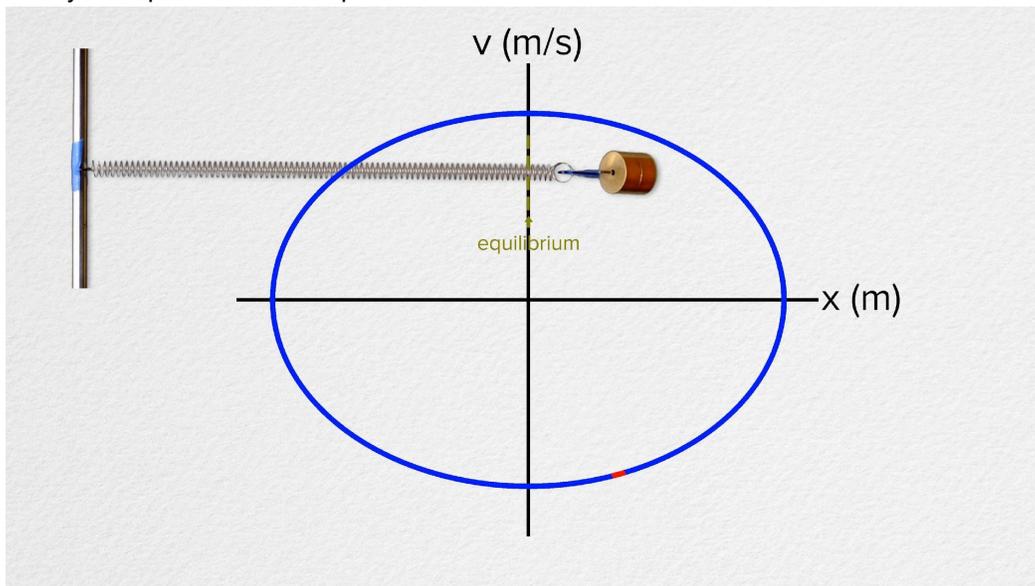
Recall that angular frequency in a mass-spring system equals the square root of spring constant over mass.

$$\omega = \sqrt{\frac{k}{m}}$$

We have solved for the velocity of a mass-spring system in terms of position:

$$\Rightarrow v(x) = \pm\omega\sqrt{A^2 - x^2}$$

This is actually an equation for an ellipse!



General equation for an ellipse centered at the origin:  $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$

<sup>1</sup> "Maximum Energy in Simple Harmonic Motion" <http://www.flippingphysics.com/shm-maximum-energy.html>

Rearranging our velocity as a function of position equation:

$$v^2 = \omega^2 (A^2 - x^2) \Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2 \Rightarrow \frac{v^2}{\omega^2} + x^2 = A^2$$

This is an equation for an ellipse where b equals 1 and rather than 1 on the right hand side we have amplitude squared.

Another interesting way to look at this is to remember we derived earlier<sup>2</sup> that:  $v_{\max} = A\omega$

Using that information, we can solve rearrange our equation for velocity as a function of position:

$$v(x) = \pm\omega\sqrt{A^2 - x^2} = \pm\omega\sqrt{\frac{A^2}{A^2} (A^2 - x^2)} = \pm\omega\sqrt{A^2 \left(\frac{A^2}{A^2} - \frac{x^2}{A^2}\right)} = \pm A\omega\sqrt{1 - \frac{x^2}{A^2}}$$
$$\Rightarrow v(x) = \pm v_{\max}\sqrt{1 - \frac{x^2}{A^2}}$$

And you can see the velocity is at its maximum magnitude when  $x = 0$  and at zero when  $x = A$ .

$$\Rightarrow v(0) = \pm v_{\max}\sqrt{1 - \frac{0^2}{A^2}} = \pm v_{\max}\sqrt{1} = \pm v_{\max}$$

$$\Rightarrow v(A) = \pm v_{\max}\sqrt{1 - \frac{A^2}{A^2}} = \pm v_{\max}\sqrt{1 - 1} = 0$$

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<sup>2</sup> "Simple Harmonic Motion Derivations using Calculus (Mass-Spring System)" <http://www.flippingphysics.com/SHM-derivation-mass-spring.html>