Advanced Math
Date____
Period____

Name_____

5B3: Geometric Series

Go to: http://www.purplemath.com/modules/series.htm. Near the top of the page, you will find links to related topics, which may be useful to you. Then answer each of the questions.

1. Describe the difference(s) between a sequence and a series. Create and label your own example of each.

A sequence is a list of numbers that follow a pattern
Ex: 1, 7, 13, 19, ...
A series is a sum of a sequence
Ex: 1+7+13

2. Describe the difference(s) between an arithmetic sequence and a geometric sequence. Create and label your own example of each.

Arithmetic sequence - always add (or subtract) to get from one term to next Ex: 5, 8, 11, ...
Geometric sequence - always multiply (or divide) to get from one term to the next Ex: 3, 6, 12, ...

3. Describe the difference(s) between an arithmetic series and a geometric series. Create and label your own example of each.

Arithmetic series - sum of arithemetic sequence (add b/t terms) Ex: 5+8+11
Geometric series - sum of geometric sequence (multiply b/t terms) Ex 3+6+12

Examples: Use Euclid's Method to find the sum. (We will look at a shortcut to find the sum of a geometric series.)

S for sum

$S = 6+18+54+\ldots+1458$

$3S = 18+54+108+\ldots+4374$ ← Multiply both sides by common ratio (r)

$-(S = 6+18+54+\ldots+1458)$ ← Rewrite original series so we can subtract eliminating many terms

$3S-S = 4374-6$ ← Do rS-S to eliminate many terms

$2S = 4374-6$

$S = \frac{4374-6}{2} = 2184$ Isolate S to find the sum

2. $\sum_{k=1}^{7}(3\cdot 2^{k-1})$ ⇒ $3\cdot2^{1-1} + 3\cdot2^{2-1} + 3\cdot2^{3-1} + \ldots + 3\cdot2^{7-1}$

$S = 3+6+12+\ldots+192$ write out series

$2S = 6+12+24+\ldots+384$ Multiply both sides by 2 (r)

$-(S = 3+6+12+\ldots+192)$

$1S = 384-3$ Do rS-S to eliminate many terms

$S = \frac{384-3}{1} = 381$ Isolate S to find the sum

Exercises.

A. **Identify each sequence as arithmetic, geometric, or neither.**
B. **Write a recursive form definition for each sequence.**
C. **Write a closed form definition for each sequence.**

1. 180, 60, 20, . . .

A. Geometric $r = \frac{1}{3}$

B. $g(n) = \begin{cases} 180 & \text{if } n=0 \\ \frac{1}{3}\,g(n-1) & \text{if } n>0 \end{cases}$

C. $G(n) = 180\left(\frac{1}{3}\right)^n$

2. 0.8, -2.4, 7.2, -21.6, . . .

A. Geometric $r = -3$

B. $g(n) = \begin{cases} .8 & \text{if } n=0 \\ -3g(n-1) & \text{if } n>0 \end{cases}$

C. $G(n) = .8(-3)^n$

3. 272, 265, 258, . . .

A. Arithmetic $d = -13$

B. $a(n) = \begin{cases} 272 & \text{if } n=0 \\ a(n-1) - 13 & \text{if } n>0 \end{cases}$

C. $A(n) = -13n + 272$

4. 0, 2, 8, 18, 32, . . . (+2, +6, +10, +14)

A. Neither

B. $b(n) = \begin{cases} 0 & \text{if } n=0 \\ b(n-1) + 4n - 2 & \text{if } n>0 \end{cases}$

C. $B(n) = 2n^2$

5. -190, -180, -170, . . .

A. Arithmetic $d = 10$

B. $a(n) = \begin{cases} -190 & \text{if } n=0 \\ a(n-1) + 10 & \text{if } n>0 \end{cases}$

C. $A(n) = 10n - 190$

6. $\frac{1}{4}, \frac{3}{8}, \frac{9}{16}, \frac{27}{32}, \ldots$

A. Geometric $r = \frac{3}{2}$

B. $g(n) = \begin{cases} \frac{1}{4} & \text{if } n=0 \\ \frac{3}{2}\,g(n-1) & \text{if } n>0 \end{cases}$

C. $G(n) = \frac{1}{4}\left(\frac{3}{2}\right)^n$

Find each sum. Use Euclid's Method or Gauss's Method when appropriate. If neither method applies, then find the sum by adding all of the terms in the series.

7. $7+(-14)+28+(-56)+\ldots 1792$

$-2S = -14 + 28 + -56 + \ldots -3584$

$S = 7 - 14 + 28 + \ldots 1792$

$-3S = -3584 - 7$

$S = \frac{-3584-7}{-3} = 1197$

8. $\sum_{k=0}^{5} 3 \cdot 4^k = 3 + 12 + 48 + \ldots 3072$

$4S = 12 + 48 + \ldots 12288$

$S = 3 + 12 + \ldots 3072$

$3S = 12288 - 3$

$S = \frac{12288-3}{3} = 4095$

9. $5+10+15+\ldots+320$

$320 = 5n$

$64 = n$

$S = \frac{64(5+320)}{2}$

$S = 10{,}400$

10. $5+10+20+\ldots+320$

$2S = 10 + 20 + 40 + \ldots 640$

$S = 5 + 10 + \ldots 320$

$S = 640 - 5$

$S = 635$

11. $\sum_{k=-3}^{3}(k+2)^3$

$= -1 + 0 + 1 + 8 + 27 + 64 + 125$

$= 224$

12. $6+3+\frac{3}{2}+\ldots\frac{3}{64}$

$\frac{1}{2}S = 3 + \frac{3}{2} + \frac{3}{4} + \ldots \frac{3}{128}$

$S = 6 + 3 + \frac{3}{2} + \ldots \frac{3}{64}$

$-\frac{1}{2}S = \frac{3}{128} - 6$

$S = \frac{\frac{3}{128} - 6}{-\frac{1}{2}} = \frac{765}{64}$

+2 +6 +10 +14

13. $0+2+8+18+32+\ldots 162$

$0 + 2 + 8 + 18 + 32 + 72 + 98 + 128 + 162$

$= 570$

14. $\sum_{x=0}^{7} 5 \cdot 3^x = 5 + 15 + 45 + \ldots 10935$

$3S = 15 + 45 + \ldots 32805$

$S = 5 + 15 + \ldots 10935$

$2S = 32805 - 5$

$S = \frac{32805-5}{2} = 16400$

15. $\sum_{x=0}^{8} 2 \cdot 1.04^x$

$1.04S = 2 \cdot 1.04^1 + 2 \cdot 1.04^2 + \ldots 2 \cdot 1.04^9$

$S = 2 + 2 \cdot 1.04^1 + \ldots 2(1.04)^9$

$.04S = 2(1.04)^9 - 2$

$S = \frac{2(1.04)^9 - 2}{.04} \approx 21.166$

16. $-10+2+14+26+\ldots 110$

$n=0$ $\quad$ $n=10$

$110 = 12n - 10$

$10 = n$

$\therefore$ 11 terms

$S = \frac{11(-10+110)}{2} = 550$

17. Because of your mathematical prowess, you have been offered two jobs.

Job A is at Gaussian Elimination. The job will pay $10,000 on the first day. The pay will increase by $1000 each day. (In other words, the job pays $10000 on day one, $11000 on day two, $12000 on day three and so on.) How much would you earn in total at Gaussian Elimination during the 30 days?

$$S = \frac{30(10{,}000 + 10000 + 1000(29)}{2} = \$735{,}000$$

Job B is at Euclidean Algorithm. The job will pay $0.01 the first day and then double the pay for the previous day after that. (In other words, the job will pay $0.01 on day one, $0.02 on day 2, $0.04 on day three, $0.08 on day and so on.) How much would you earn in total at Euclidian Algorithm during the 30 days?

$$S = .01 + .02 + \ldots .01(2)^{29}$$

$$2S = .02 + .04 + \ldots .01(2)^{30}$$

$$-\ S = .01 + .02 + \ldots .01(2)^{29}$$

$$S = .01(2)^{30} - .01 = \$10{,}737{,}418.23$$

Which job is more lucrative and by how much?

Euclidean Algorithm by $10,002,418.27

18. People often invest some money each month or year into a retirement account. Suppose you plan to invest $3600 into a retirement account. Your first investment will take place at age 25 and your last investment will take place at age 65. What is your retirement account balance at age 65 under each of the following conditions? (Your investment made at age 25 will earn interest for 40 years, while the investment made at age 65 will earn interest for 0 years for this question.)

A. Your investment earns 3% APR compounded annually.

$$S = 3600(1.03)^{40} + 3600(1.03)^{39} + \ldots 3600$$

$$1.03S = 3600(1.03)^{41} + 3600(1.03)^{40} + \ldots 3600(1.03)$$

$$-\ S = 3600(1.03)^{40} + 3600(1.03)^{39} + \ldots 3600$$

$$.03S = 3600(1.03)^{41} - 3600$$

$$S \approx \$283{,}187.87$$

B. Your investment earns 6% APR compounded annually.

$$1.06S = 3600(1.06)^{41} + 3600(1.06)^{40} + \ldots 3600$$

$$-\ S = 3600(1.06)^{40} + \ldots 3600$$

$$.06S = 3600(1.06)^{41} - 3600$$

$$S \approx \$594{,}171.66$$

C. Your investment earns 9% APR compounded annually.

$$S = \frac{3600(1.09)^{41} - 3600}{.09} \approx \$1{,}329{,}450.71$$

Selected Exercise Answers:

7. 1197 8. 4095 9. 10400 10. 635 11. 224 12. 765/64 13. 570 14. 16400 15. about 21.166 16. 550