Andre Mas | Research Statement

The radiative transport equation (RTE) describes the propagation of photons through a medium while accounting for absorption, emission, and scattering. Each of these effects depend on the optical properties of the medium, as well as the travel direction and energy of photons. Solving the RTE when the medium's properties are known is called the *forward problem*. In many imaging applications, the corresponding *inverse problem* is of greater interest: given boundary measurements of solutions to the RTE, can one recover the underlying optical properties of the medium? Such problems are mathematically challenging because small errors in the measured data may lead to large errors in reconstruction—a phenomenon known as ill-posedness. Overcoming ill-posedness requires careful mathematical formulation and often the development of specialized regularization techniques.

My research focuses on two-dimensional models of X-ray and optical tomography under the so-called "single-scattering approximation." The goal is to recover both the attenuation and scattering coefficients of a heterogeneous medium, a problem motivated by medical and homeland-security applications. Simultaneous recovery of both coefficients is considerably harder than reconstructing either one alone and remains an active area of research. Unlike traditional X-ray tomography, which neglects scattering, incorporating scattering yields a more complete description of the medium and enables local inversion formulas that may require fewer detector measurements. This has the potential benefit of reducing radiation exposure in a medical context while improving reconstruction quality.

In the setting of single-scattering tomography, detector measurements are naturally related to the unknown coefficients through a mathematical object known as the star transform. Inversion of the star transform is the central mechanism by which recovery of attenuation and scattering coefficients may be achieved. My research focuses on developing new inversion formulas for the star transform using techniques from applied harmonic analysis and variational calculus, with the goal of making these reconstructions more accurate and efficient. In particular, I am interested in extending these methods to the case of energy-dependent attenuation, in which energy loss due to scattering is modeled by the well-known Compton scattering formula.

References

- Fan Zhao, John C Schotland, and Vadim A Markel. Inversion of the star transform. Inverse Problems, 30(10), October 2014.
- R Krylov and A Katsevich. Inversion of the broken ray transform in the case of energy-dependent attenuation. Physics in Medicine and Biology, 60(11), June 2015.
- Gaik Ambartsoumian. Generalized radon transforms and imaging by scattered particles: broken rays, cones, and stars in tomography. 2023.