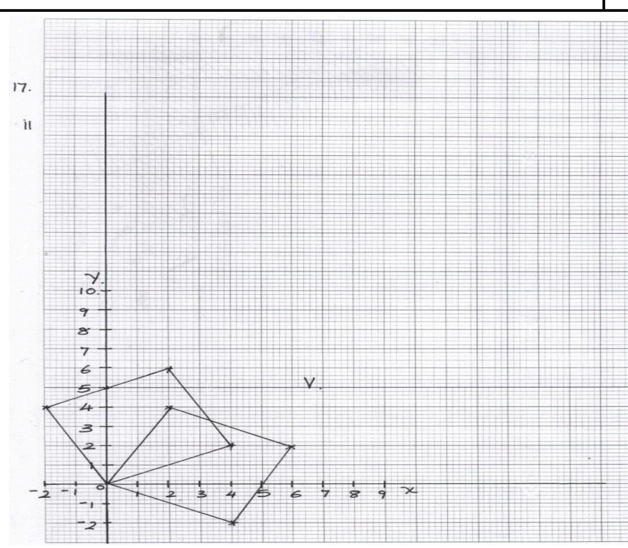
2. *Matrices and Transformations*

1	a) (i)		
	(ii)	B_1	Square S
	$\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}\begin{pmatrix}0 & 2 & 2 & 0\\0 & 0 & 2 & 2\end{pmatrix}$	M_1	drawn
	$=\begin{pmatrix}0 & 4 & 2 & -2\\0 & 2 & 6 & 4\end{pmatrix}$		
	$T \Rightarrow (0,0), (4,-2), (2,6), (-2,4)$	A_1 B_1	✔coordinates given
	(iii) $\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\begin{pmatrix}0 & 2 & 2 & 0\\0 & 0 & 2 & 2\end{pmatrix}=\begin{pmatrix}0 & 4 & 6 & 2\\0 & -2 & 2 & 6\end{pmatrix}$	B_1 B_1	Square T drawn
	$U \Rightarrow (0,0), (4,-2), (6,2), (2,4)$		✔coordinates (implied)
	b) (i)	B_1	Square U
	$\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}=\begin{pmatrix}5 & 0\\0 & 5\end{pmatrix}$		drawn
	$\begin{pmatrix}5 & 0\\0 & 5\end{pmatrix}\begin{pmatrix}0 & 2 & 2 & 0\\2 & 0 & 2 & 2\end{pmatrix}=\begin{pmatrix}0 & 10 & 10 & 0\\0 & 0 & 10 & 10\end{pmatrix}$	B_1 B_1 B_1	
	$V \Leftarrow (0,0), (10,0), (10,10), (0,10)$		
	(ii) Enlargement center (0,0) s.f = 5		✔coordinates (implied) Square V drawn
			
		10	

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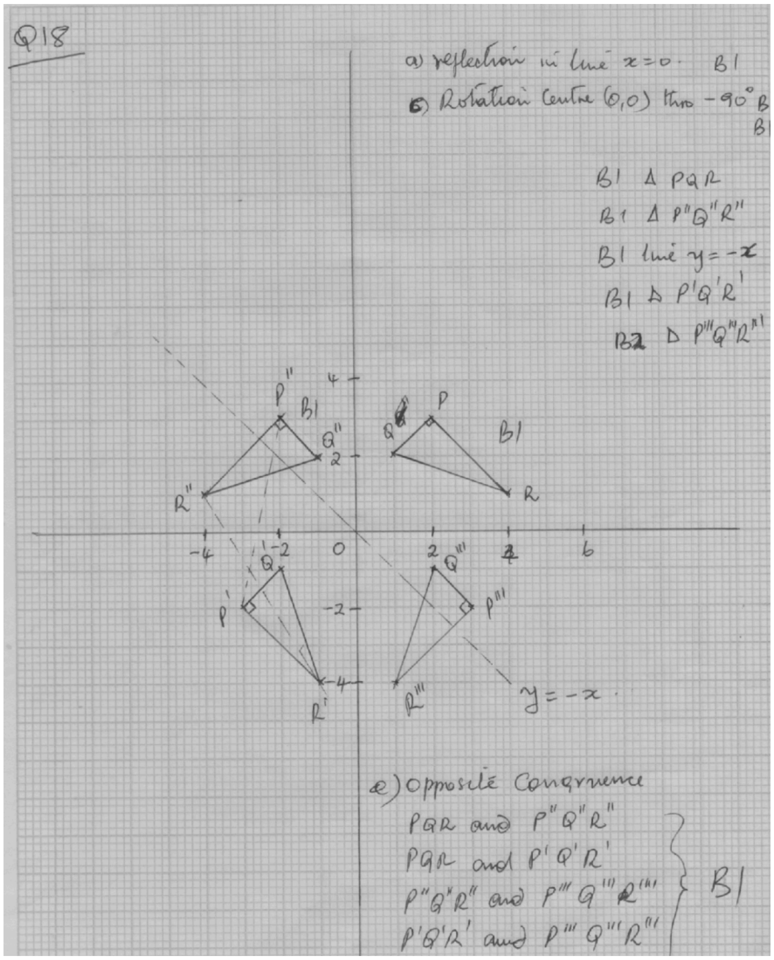
<tr><td>2.</td><td>

Total 10</td><td></td><td></td></tr>
<tr><td></td><td></td><td>10</td><td></td></tr>
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1. *a) B (4,-5), C (3,6 ½)*

Δ ABC drawn

Δ ABC drawn

a) ii) Shear maps

1

I (1, 1½)

Matrix = 1 0

1 ½

1 1 1

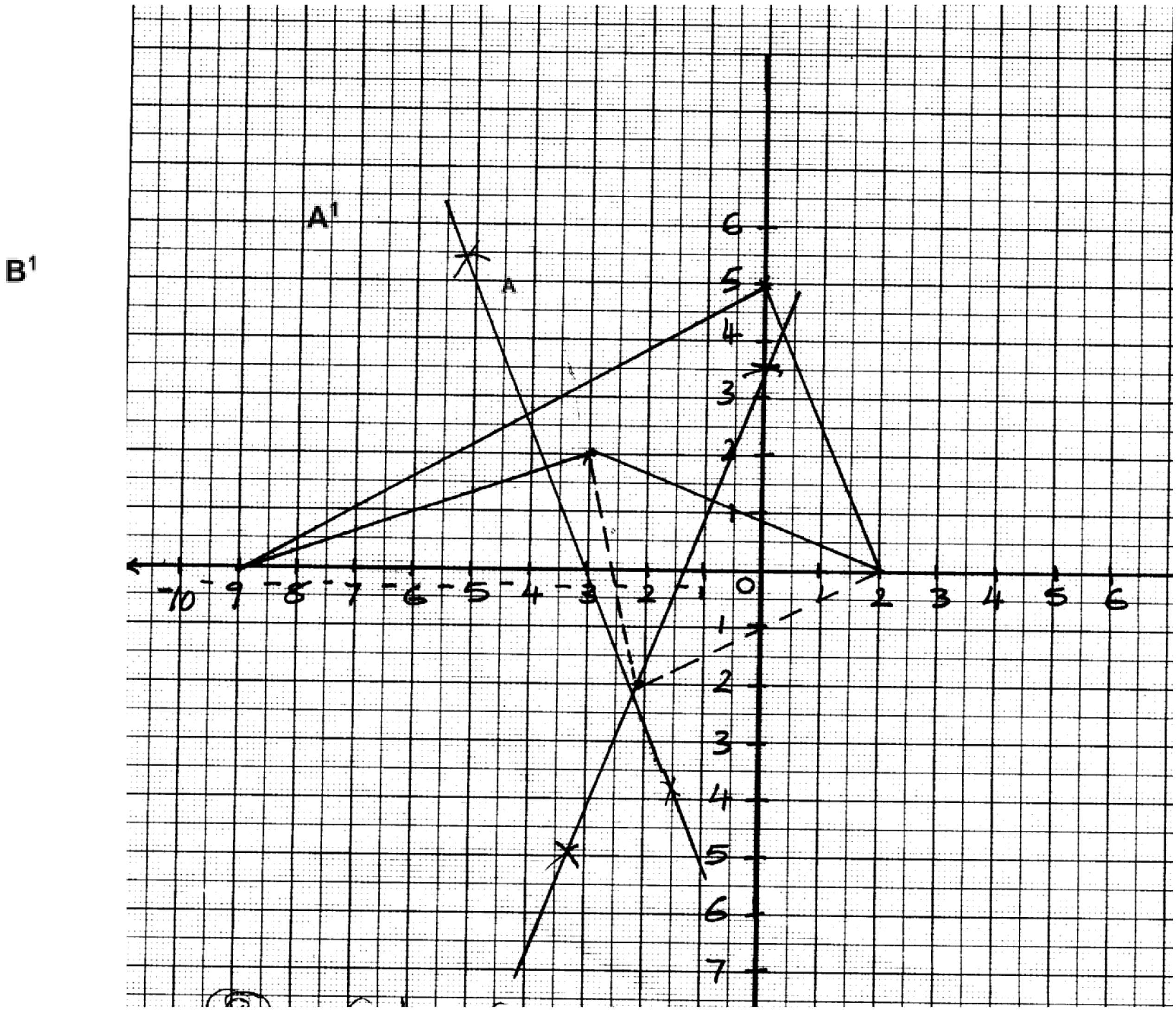
b) i) $\begin{pmatrix} \\ \\ \end{pmatrix} \begin{array}{ccc} A & B & C \end{array} \begin{pmatrix} \\ \\ \end{pmatrix}$

$= \begin{array}{ccc} A^{11} & B^{11} & C^{11} \end{array} \begin{pmatrix} \\ \\ \end{pmatrix}$

$\Delta\ A^{11}\ \ B^{11}\ \ C^{11}\ \ D^{11}$ drawn

ii) Half turn about (0,0)

2.

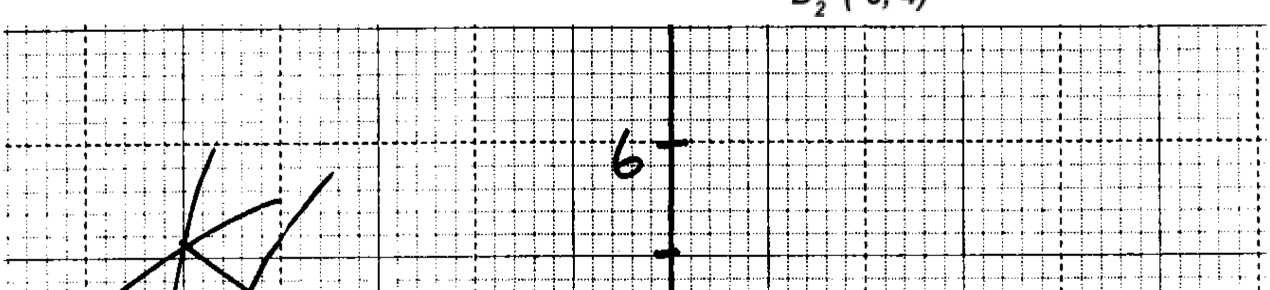


(a) Centre (-2, -2) 90º
(b) A11 (-2 , -4) , B11 (0, 9)
(c) Half-turn about the centre (0, 2)

3.

A_1 (-2, -1)
B_1 (-2, -6)
C_1 (4, -6
D_1 (-4, -1)
A_2 (-1, 2)
B_2 (-2, -6)
C_2 (-6, 4)
D_2 (-6, 4)

(b)



(c) (i) U - - positive three-quarter turn about the origin

(ii)UT – Reflection I the line x = 0

(d) IdetI = I2.5 x -2 – 1x 0 I= 5

∴Area = 5x(5x2) = 20sq. units

4. (a)

b) Centre (-2, 4)

Angle + 90°

5. $P(5,-3)$ $P^1\ (2, -5)$

$+ \quad =$

$=$

$R^1 = \quad +$

$=$

$P^1R^1 = \quad -$

$=$

Mag. = 7units

6. $A^1 = (0+1, -1-2) = (1, -3)$

$B^1 = (4 + 1)\ , 3-2) = (4, 1)$

$C^1 = (\ 2 +1, 2-2) = (3-0)$

Matrix

$=$

A^{11} (3, -9) B (15, 3) C (9,0)

Determinant (0-9) =-9

Area = 9x24 = 216cm^2

$\underline{-15a+3b=5}$ $\underline{15c+3d=1}$

$-48b=0$ $-48d=-16$

$b=0$ $d=\frac{1}{3}$

$a=\frac{1}{3}$ $c=0$

matrix

7. Scale u

ΔABC drawn B_1

$\Delta A_1B_1C_1$ drawn B_1

A, (6, -1), B(7, 2) C, (4, 4) B1

Line x = 4 L_1

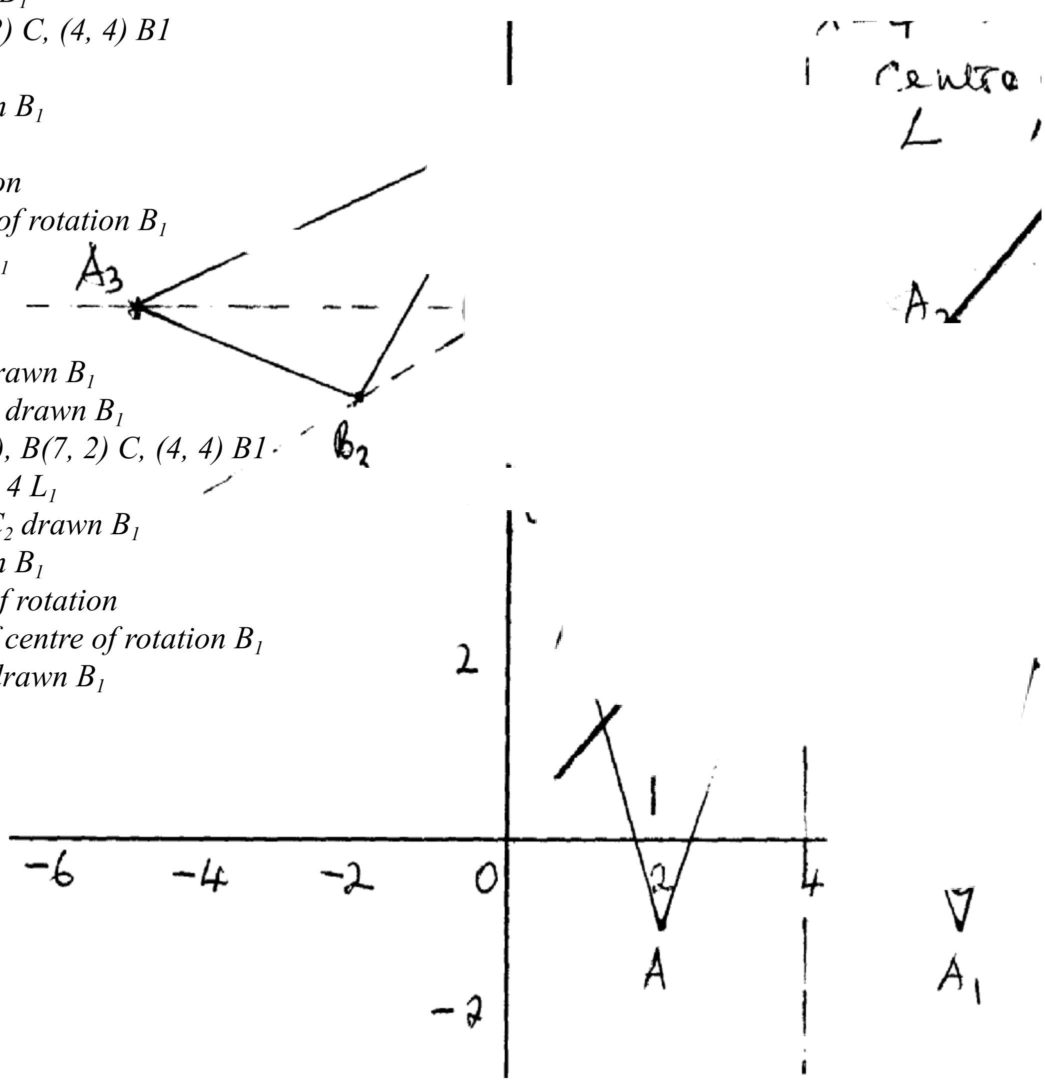
$\Delta A_2\ B_2\ C_2$ drawn B_1

Two seen B_1

Centre of rotation

Angle of centre of rotation B_1

$A_3B_3C_3$ drawn B_1



Scale used S_1

ΔABC drawn B_1

$\Delta A_1B_1C_1$ drawn B_1

A, (6, -1), B(7, 2) C, (4, 4) B1

Line x = 4 L_1

$\Delta A_2\ B_2\ C_2$ drawn B_1

Two seen B_1

Centre of rotation

Angle of centre of rotation B_1

$A_3B_3C_3$ drawn B_1

8. (a) P(6, -2)

$X^1 = 6 - 3(-2) = 12$

$Y^1 = 2(6) = 12$

$(X^1, Y^1) = (12, 12)$

(b) (i) $A^1(3, 4)$
(ii) $B^1 (3, 2)$
$C^1 (1, 4)$
$D^1(4, 3)$

(c) (i) $\begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} A^1 & B^1 & C^1 & D \\ 3 & 3 & 1 & 4 \\ 4 & 2 & 4 & 5 \end{pmatrix}$

$= \begin{pmatrix} & \\ & \end{pmatrix}$

$A^{11} (-5, 4)$, $B^{11}(-1, 2)$, $C^{11}(-7, 4)$ and $D^{11}(-6, 5)$

(ii) A stretch with y-axis invariant and a sketch factor (3)

$2h = 6$
$h = 3$

$-5a + 4b = 4$ $\quad$ $-5c + 4d = -3$
$\underline{- a + 2b = 2}$ $\quad$ $\underline{-c + 2d = 3}$
$-5a + 4b = 4$ $\quad$ $-5c + 4d = -3$
$\underline{-a + 4b = 4}$ $\quad$ $\underline{-c + 4d = -6}$
$-4a = 0$ $\quad$ $- 4c = 3$
$a = 0$ $\quad$ $c = - ¾$
$b = 1$ $\quad$ $d = {}^{15}/_{8}$

9. (a) $X1(5, -1)$ $y_1(7, -1)$ $Z_1 (-2, 2)$
xyz & $x_1y_1z_1$ well drawn

(b) 1-3 xyz x1y1z1
$X_2(2, 10)$ $y_2(2, 14)$
$X_2y_2Z_2$ well drawn

(c) $\begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} & & \\ -1, & -1, & 2 \end{pmatrix} \begin{pmatrix} & & \\ & & \end{pmatrix}$

$\begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0, & 1 \end{pmatrix} \begin{pmatrix} & \\ & \end{pmatrix}$

(d)) Area of $\Delta X_2y_2Z_2$
$= 4x15 = 60cm^2$

10. $\begin{pmatrix} & \\ & \end{pmatrix} \begin{matrix} 2 & 4 & 4 & 2 \\ 1 & 1 & 4 & 4 \end{matrix} = \begin{matrix} 7 & 14 & 14 & 8 \\ 8 & 7 & 16 & 16 \end{matrix}$

$2a + b = 8$
$\underline{4a + b = 14}$
$-2a = -6$
$6 + b -= 8$
$b = 2$
$\therefore 6 + b = 8$
$b=2$
$2c + d = 7$
$\underline{4c + d = 7}$
$-2c = 0$
$c = 0$
$d = 7$
$\begin{pmatrix} 3 & 2 \\ 0 & 7 \end{pmatrix}$

$\therefore$

- it is an enlargement with scale factor 3 with centre (-1, -2)

(c) + =

$a + 8 = 7 \qquad 7 + b = 9$

$a = -1 \qquad b = 2$

$\therefore T =$

11. a) *ABCD drawn* B_1

Name – Parallelogram B_1

b) $A^1B^1C^1D^1$ *drawn* B_1

Attempt to joining any two points and bisecting. B_1

Description – Rotation + 90^0. B_1 *or quarter turn about (0,0)*

c) $A^{11}B^{11}C^{11}D^{11}$ *drawn.* B_1

Description – Enlargement centre (0, 0) Scale factor –Z. B_1

d) $A^{111}B^{111}C^{111}D^{111}$ *– drawn.* B_1

Attempt to reflect. B_1

Coordinates

A^{111} = 9-2, 4) $\qquad$ C^{111} = (-8, 4) B_1 *All correct*

B^{111} = (-6, 0) $\qquad$ D^{111} (-4, 8)

12.

$B^{|}$(-2, 6) $\qquad$ $C^{|}$(6, -16)

-2 = 6

6 -6

10, 10) C'' (18, -6)

MN

=

=

$p\text{-}1 = \frac{1}{-12}$

13. $Det = 2 - 6$

$= -4$

$A.S.F = 4$

$\frac{25.6}{x} = 4$

$x = 6.4cm^2$

Area of $\Delta ABC = 6.4cm^2$

14. $T + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$T = \begin{pmatrix} 4 - 2 \\ 0 + 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\therefore \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$Q\ (1,6)$

16. $5x^2 + 6 = {}^{110}/_{10}$

$5x^2 + 6 = 11$

$x^2 = 1$

$x = \pm 1$