

Teacher Facilitation Guide

Grade 6 **Math**

Unit 1 Lesson 1

[Unit Plan](#)

[Scope and
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[Instructional
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Grade 6, Unit 1, Lesson 1: Tiling the Plane

[Skyline Playlist](#)

LESSON OVERVIEW: Students start the first lesson of the school year by recalling what they know about **area** (note that students studied the areas of rectangles with whole-number side lengths in grade 3 and with fractional side lengths in grade 5). The mathematics they explore is not complicated, so it offers a low threshold for entry. The lesson does, however, uncover two important ideas:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- The area of a **region** does not change when the region is decomposed and rearranged.

At the end of this lesson, students are asked to write their best definition of area. It is important to let them formulate their definition in their own words. For English learners, it is especially important that they be encouraged to use their own words and also to use words of their peers. In the next lesson, students will revisit the definition of area as the number of square units that cover a region without gaps or overlaps.

As the first set of problems in a problem-based curriculum, students will also begin their year-long work on making sense of problems and persevere in solving them (MP1). This opening lesson leaves space for teachers to begin setting classroom routines and their expectations for mathematical discourse (MP3).

In all of the lessons in this unit, students should have access to their geometry toolkits, which should contain tracing paper, graph paper, colored pencils, scissors, and an index card. Students may not need all (or even any) of these tools to solve a particular problem. However, to make strategic choices about when to use which tools (MP5), students need to have opportunities to make those choices. Apps and simulations should supplement rather than replace physical tools.

Notes on terminology. In these materials, when we talk about a figure such as a rectangle, triangle, or circle, we usually mean the boundary of the figure (e.g., the sides of the rectangle), not including the region inside. However, we also use shorthand language such as “the area of a rectangle” to mean the “the area of the region inside the rectangle.” The term shape could refer to a figure with or without its interior. Although the terms figure, region, and shape are used without being defined precisely for students, help students understand that sometimes our focus is on the boundary (which in this unit will always be composed of black line segments), and sometimes it is on the region inside (which in this unit will always be shown in color and referred to as “the shaded region”).

Enduring Understandings: Shapes can be decomposed and rearranged to more easily calculate their area.	Learning Objectives/Targets: • Compare (orally) areas of the shapes that make up a geometric pattern. • Comprehend that the word “area” (orally and in writing) refers to how much of the plane a shape covers. Student-Facing: • I can explain the meaning of area.
Essential Questions: (Student Friendly Lang.) What is area?	Technology/Materials/Resources: • Student E-Workbook • Student E-Workbook in Spanish • Student Task Statement • Student Task Statement in Spanish • Practice Problems • Practice Problems in Spanish • Cool Down • Cool Down in Spanish • Tracking paper • Graph paper • Colored pencils • scissors • Fluency Games • Grade 6 Glossary
Learner Relevance: • This lesson uses patterns in a mathematical context rather than a real-world context. • This lesson playlist includes Learning to Learn opportunities meant to assist students in acquiring greater mathematical identity and agency. These metacognitive reflection points and discussion prompts help increase relevance for students and increase exposure to new ideas and strategies for solving problems.	Lesson Priority Vocabulary: • Area • Region

- For students with an Individual Education Program, refer to the IEP to learn what Specially Designed Instruction (SDI) is required to adapt the lesson.

Necessary Background Knowledge: In Grade 3, students worked with shapes and their attributes. They partitioned shapes into parts with equal areas and expressed the area of each part as a unit fraction of the whole. Recognizing the relationships between the partitioned shapes and the larger shapes are necessary background knowledge for this lesson.

Content Standards:

Addressing: CC.3.G.1 Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals that do not belong to any of these subcategories.

CC.3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Building Toward: CC.6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Procedures & Routines: 50 minutes

- Warm-Up: Anticipate, Monitor, Select, Sequence, Connect
Think Pair Share
MLR2: Collect and Display
- Activity 1.1: Which One Doesn't Belong?

Formative Assessments:

- This lesson has a Cool Down and Practice Problems at the end to assess student understanding of the lesson objectives.
- [Pre-Unit Assessment \(Checkpoint\)](#)
- [Pre-Unit Assessment \(Checkpoint\) in Spanish](#)
- [Unit 1 End-of-Unit Assessment](#)
- [Unit 1 End-of-Unit Assessment in Spanish](#)

Opportunities for Discourse:

- Warm-Up: Class and group discussion.
- Activity 1.1: Partner discussion



ENGLISH LEARNERS

WIDA Language Standards

WIDA Standard 1: Social and instructional Language Standards

Key Uses addressed in this lesson:

Discuss: See oral language in the Can Do Descriptors by Language Domain, Proficiency Level, and Key Use of Language.

- Students answer questions about the solution strategies as they are presented.

WIDA Standard 3: The Language of Mathematics

Key Uses addressed in this lesson:

Discuss: See oral language in the Can Do Descriptors by Language Domain, Proficiency Level, and Key Use of Language.

Language Objective

Students will use speaking and listening skills to discuss representing and solving equations in one variable.

Level 1

Level 2-3

Level 4-5

MPI

Contribute to conversations by sharing own work when representing and solving equations in one variable with tape diagrams, using MLR2: Collect and Display, drawings, equations, word bank, and partners.

Support ideas with examples when representing and solving equations in one variable with tape diagrams, using MLR2: Collect and Display, drawings, equations, and partners.

Build on the ideas of others when representing and solving equations in one variable with tape diagrams, using drawings, equations, and partners.

Supports

Mathematics Language Routines (MLR2: Collect and Display)
Mathematical Representations (drawings, equations)
Vocabulary (Word banks)
Grouping Structures

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Grouping Structures

Mathematical Representations (drawings, equations)
Grouping Structures

To determine what your students can do, go [here](#). This resource assists teachers in determining what students can do within the Communicative Purposes called Key Uses.

**ENGLISH LEARNERS****Support Mini-lessons**

6U1L1-3 Mini Lesson A: Compose and Decompose (This mini lesson can be used between Lessons 1-3)

- 6U1L1-3 Mini Lesson A: Compose and Decompose [Playlist](#)
- 6U1L1-3 Mini Lesson A: Compose and Decompose [Spanish Playlist](#)

Learning Objective: I can describe what the words compose and decompose mean and identify when shapes are composed and decomposed.

Language Objective: Explain and justify why representations reflect composing a shape or decomposing a shape, by using supports such as virtual manipulatives, visuals, and working with a small group.

Materials:

- [Mini Lesson Teacher Facilitation Guide](#)
- [Blackline Master: Compose and Decompose](#)
- [Blackline Master in Spanish: Compose and Decompose](#)
- [Blackline Master: Puzzle \(optional\)](#)
- [Blackline Master in Spanish: Puzzle \(optional\)](#)

**SOCIAL EMOTIONAL LEARNING****ISBE Social and Emotional Learning Standards**

- 2C. Use communication and social skills to interact effectively with others.
 - 2C.3b. Demonstrate cooperation and teamwork to promote group effectiveness.
 - Activity: The lesson cycle incorporates an “explore” and “discuss” component within each lesson. Student collaboration, sharing of unique approaches and viewpoints, and connecting student approaches to the underlying mathematics is a daily occurrence. View the Course Overview of Instructional Routines for further guidance.
- 2B. Recognize individual and group similarities and differences.
 - 2B.2a. Identify differences among and contributions of various social and cultural groups.

- Activity: In this lesson, students explore tilings. To set the context, students learn that tilings and geometric patterns have a rich history spanning across numerous cultures and professions including artists, architects and crystallographers.

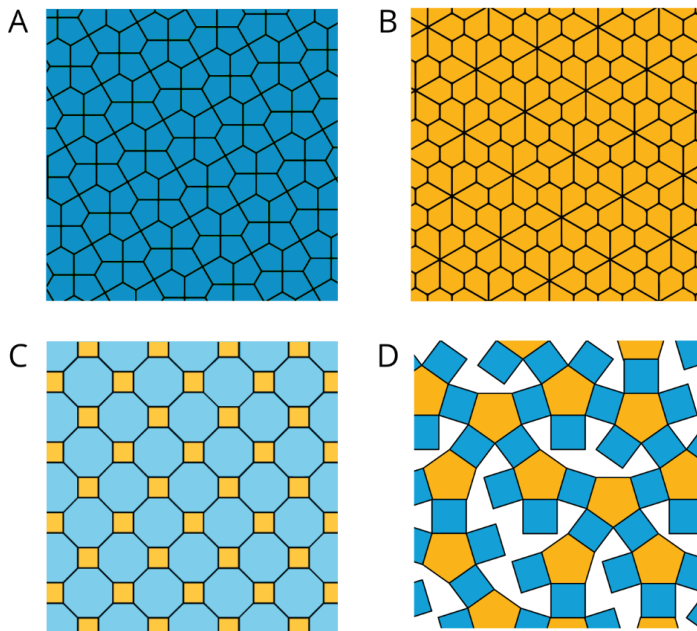
Assignments & Activities

Warm-Up: Which One Doesn't Belong: Tilings

Launch: Arrange students in groups of 2–4. Display the four patterns for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed one pattern that does not belong and can explain why. Encourage them to think of more than one possibility. When the minute is up, give students 2 minutes to share their response with their group, and then together find at least one reason, if possible, that each pattern doesn't belong.

Student Facing Task Statement:

Which pattern doesn't belong?



Possible Responses:

Answers vary. Sample responses:

- A: It doesn't have any yellow. Groups of four pentagons make hexagonal shapes that interlock without gaps.

- B: It doesn't have any blue. Groups of six pentagons make flower-like shapes that interlock without gaps.
- C: It doesn't have any pentagons. It has octagons and squares. The polygons that make up the patterns are very different in size.
- D: It has gaps between the shapes. Not all of the colored polygons meet another colored polygon on all sides. It has white (or non-filled) shapes that are more complex than other colored shapes. It is the only one where all the polygons have the same side length.

Warm-Up Synthesis:

After students shared their observations in groups, invite each group to share one reason why a particular figure might not belong. Record and display the responses for all to see. After each response, poll the rest of the class to see if others made the same observation.

Since there is no single correct answer to the question of which pattern does not belong, attend to students' explanations, and make sure the reasons given are correct. Prompt students to explain the meaning of any terminology they use (names of polygons or angles, parts of polygons, area, etc.) and to substantiate their claims. For example, a student may claim that Pattern D does not belong because its polygons all have the same side length. Ask how they know that is the case, and whether that is true for the white (or non-filled) polygon.

Explain to students that covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps is called "tiling" the plane. Patterns A, B, and C are examples of tiling. Tell students that we explore more tilings in upcoming activities.

Activity 1.1: More Red, Green, or Blue?

Purpose:

Instructional Routines	• Anticipate, Monitor, Select, Sequence, Connect • Think Pair Share • MLR2: Collect and Display
CCSS: Building Towards	• CC.6.G.1

This activity asks students to ML, with the aim of supporting two big ideas of the unit:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- A region can be decomposed and rearranged without changing its area.

Students are likely to notice that in each pattern:

- The same three polygons (triangles, rhombuses, and trapezoids) are used as tiles.
- The entire tiling pattern is composed of these hexagons.
- The shapes are arranged without gaps and overlaps, but their arrangements are different.
- A certain set of smaller tiles form a larger hexagon. Each hexagon has 3 trapezoids, 4 rhombuses, and 7 triangles.

Expect some students to begin their comparison by counting each shape, either within a hexagon or the entire pattern. To effectively compare how much of the plane is covered by each shape, however, they need to be aware of the relationships between the shapes. For example, two green triangles can be placed on top of a blue rhombus so that they match up exactly, which tells us that two green triangles cover the same amount of the plane as one blue rhombus. Monitor for such an awareness. (It is not necessary for students to use the word “area” in their explanations. At this point, phrases such as “they match up” or “two triangles make one rhombus” suffice.)

If students are not sure how to approach the questions, encourage them to think about whether any tools in their geometry toolkits could help. (For example, they could use tracing paper to trace entire patterns or certain shapes to make comparison, or use a straightedge to extend lines within the pattern. Some students may be inclined to cut out and compare the shapes.) Pattern tiles, if available, can be offered as well.

During the partner discussion, monitor for groups who discuss the following ideas so that they can share later, in this sequence:

- Relationships between two shapes: E.g., 2 triangles make a rhombus, and 3 triangles make a trapezoid.
- Relative overall quantities: E.g., there are 64 green triangles, 32 blue rhombuses (which have the same area as 64 triangles), and 24 red trapezoids (which have the same area as 72 triangles), so there is more red.
- Relative quantities in a hexagon: E.g., in each hexagon there are 7 green triangles, 4 rhombuses (which have the same area as 8 triangles), and 3 trapezoids (which have the same area as 9 triangles).

Classrooms using the digital activity have the option for students to use an applet that allows for the pattern to be isolated and also framed. This might assist students in focusing on how many of each shape comprise the pattern.

Launch:

Support for **ENGLISH LEARNERS**

Conversing, Speaking, Listening: Math Language Routine 2, Collect and Display.

This is the first time Math Language Routine 2 is suggested as a support in this course. In this routine, the teacher circulates and listens to students talk while writing down the words, phrases, or drawings students produce. The language collected is displayed visually for the whole class to use throughout the lesson and unit. Generally, the display contains different examples of students using features of the disciplinary language functions, such as interpreting, justifying, or comparing. The purpose of this routine is to capture a variety of students' words and phrases in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.

Design Principle(s): Support sense-making

How It Happens:

1. After assigning students to work on Pattern A or B, circulate around the room and collect examples of language students are using to compare areas of polygons. Focus on capturing a variety of language describing the relationship between the size of two shapes, comparing overall quantities of shapes to equivalent areas of other shapes, and comparing relevant quantities in a hexagon. Aim to capture a range of student language that includes formal, precise, complete ideas and informal, ambiguous, and partial ideas. Plan to publicly update and revise this display throughout the lesson and unit.
If pairs are stuck, consider using these questions to elicit conversation: "How many green triangles, blue rhombuses, and red trapezoids are in each pattern?", "Three triangles is equivalent to how many trapezoids?", and "Which shapes make up a hexagon?"
If using the applet, have pairs use the applet together. Check that students focus on how many of each shape comprise the pattern by hiding, moving, and turning the shapes.
2. Create a display that includes visual representations of the words and phrases collected. Group language about Pattern A on one side of the display and language about Pattern B on the other side.
3. Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and then have students move on to discussing other aspects of the activity. Continue to publicly update and revise the display throughout the lesson and unit.

Support for Students with Disabilities**UNIVERSAL DESIGN FOR LEARNERS**

Conceptual Processing: Manipulatives. For students who benefit from hands-on activities, provide pattern tiles (or pre-cut an extra copy of each pattern) for students to compare the shapes.

Arrange students in groups of 2. Ask one partner to analyze Pattern A and the other to analyze Pattern B. Tell students that their job is to compare the amount of the plane covered by each shape in the pattern.

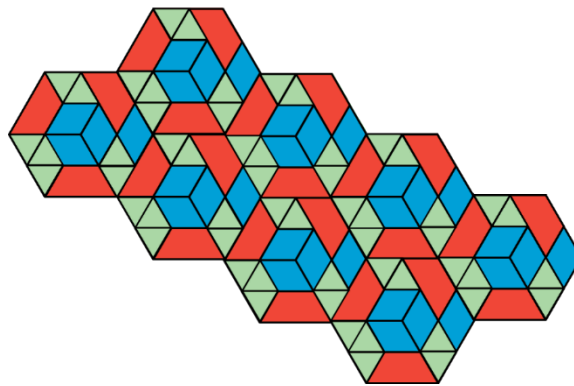
Before students begin, introduce students to the geometry toolkits, and explain that they can use the toolkits for help, if needed. Give students 7–8 minutes of quiet think time. Then, ask students to share their responses with their partners and follow with a whole-class discussion.

Student Facing Task Statement:

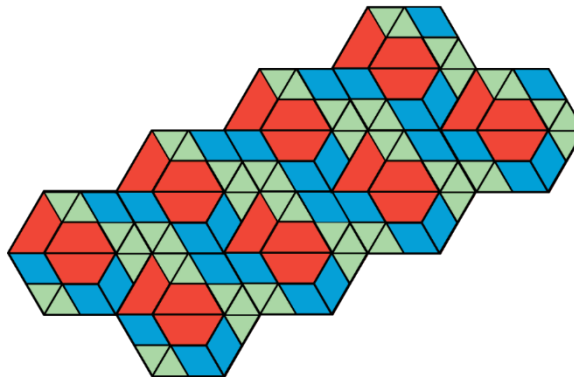
Your teacher will assign you to look at Pattern A or B.

In your pattern, which shapes cover more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

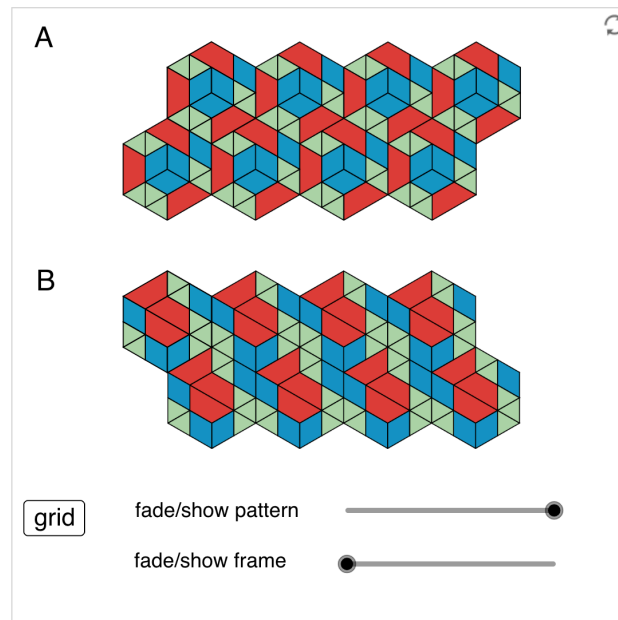
Pattern A



Pattern B



You may use this applet to help. Explore what you can see or hide, and what you can move or turn.



Possible Response:

In both Patterns A and B, more of the plane is covered by red trapezoids than green triangles or blue rhombuses. Possible explanations:

- Patterns A and B are each made of 56 green triangles, 32 blue rhombuses, 24 red trapezoids.
 - One red trapezoid covers the same amount of the plane as 3 green triangles, so 24 red trapezoids cover the same amount of the plane as 72 green triangles, which are more than the 56 green triangles.
 - Each blue rhombus covers the same amount of the plane as 2 green triangles, so the 32 rhombuses cover the same amount of the plane as 64 green triangles, which are also more than the 56 green triangles.
- Each pattern is composed of 8 hexagons. In each hexagon there are 3 red trapezoids, 4 blue rhombuses, and 7 green triangles.
 - Two red trapezoids can be arranged into a small hexagon. Three rhombuses can also be arranged into the same small hexagon. This means 2 trapezoids cover the same amount of the plane as 3 rhombuses.
 - Each large hexagon has 3 red trapezoids and 4 blue rhombuses. Since 2 trapezoids are equal to 3 rhombuses, we can just compare 1 trapezoid and 1 rhombus. We can see that 1 red trapezoid covers more of the plane than 1 rhombus.

- Each large hexagon has 3 red trapezoids and 7 green triangles. One trapezoid covers the same amount of the plane as 3 triangles, so 3 trapezoids cover the same amount of the plane as 9 triangles, which are more than 7 green triangles.

Anticipated Misconceptions:

Students may say more of the area is covered by the color they see the most in each image, saying, for example, “It just looks like there is more red.” Ask these students if there is a way to prove their observations.

Students may only count the number of green triangles, red trapezoids, and blue rhombuses but not account for the area covered by each shape. If they suggest that the shape with the largest number of pieces covers the most amount of the plane, ask them to test their hypothesis. For example, ask, “Do 2 triangles cover more of the plane than 1 trapezoid?”

Students may not recall the terms trapezoid, rhombus, and triangle. Consider reviewing the terms, although they do not need to know the formal definitions to work on the task.

Activity 1.1 Synthesis:

Select previously identified students or groups to share their answers and explanations. Sequence the explanations in the order listed in the Activity Narrative. To clarify the idea of comparing shapes by placing them on top of one another and seeing if or how they match, consider demonstrating using the digital applet.

Then, make it explicit that when we ask, “Which type of shape covers more of the plane?” We are asking them to compare the **areas** covered by the different types of shapes. To recast the comparisons of the shapes in terms of area, ask questions such as:

- “How does the area of the trapezoid compare to the area of the triangle?” (The area of the trapezoid is three times the area of the triangle.)
- “How does the area of the rhombus compare to the area of the triangle?” (The area of the rhombus is twice the area of the triangle.)
- “Is it possible to compare the area of the rhombuses in Pattern A and the area of the triangles in Pattern B? How?” (Yes, we can count the number of rhombuses in A and the number of triangles in B. Because 2 triangles have the same area as 1 rhombus, we divide the number of triangles by 2 to compare them.)

Lesson Synthesis:

In this lesson, we have started to reason about what it means for two shapes to have the same **area**. We started doing mathematics and thinking about tools that can help us. Ask students:

- “What are some of the tools in the geometry toolkit and what are they used for?”
- “Draw two shapes that you know do not have the same area. How can you tell?”

Tell students that we will continue to think about area, to do and talk about mathematics, and to learn to use tools strategically.

Support for  **ENGLISH LEARNERS**
Design Principle(s): Support sense-making

How It Happens:

1. After assigning students to work on Pattern A or B, circulate around the room and collect examples of language students are using to compare areas of polygons. Focus on capturing a variety of language describing the relationship between the size of two shapes, comparing overall quantities of shapes to equivalent areas of other shapes, and comparing relevant quantities in a hexagon. Aim to capture a range of student language that includes formal, precise, complete ideas and informal, ambiguous, and partial ideas. Plan to publicly update and revise this display throughout the lesson and unit.

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If using the applet, have pairs use the applet together. Check that students focus on how many of each shape comprise the pattern by hiding, moving, and turning the shapes.

2. Create a display that includes visual representations of the words and phrases collected. Group language about Pattern A on one side of the display and language about Pattern B on the other side.
3. Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and then have students move on to discussing other aspects of the activity. Continue to publicly update and revise the display throughout the lesson and unit.

Cool Down: Finish the Diagrams

The purpose of this cool down is to check how students are thinking about area after engaging in the activities. While the task prompts students to reflect on the work in this lesson, ideas about area from students' prior work in grades 3–5 may also emerge. Knowing the range of student thinking will help to inform the next day's lesson.

Student-Facing Task Statement:

Think about your work today, and write your best definition of area.

Possible Responses:

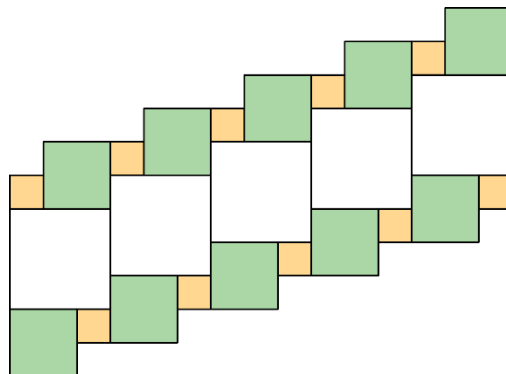
Answers vary. Sample responses:

- The amount of space inside a two-dimensional shape
- The measurement of the inside of a polygon
- The number of square units inside a shape

Lesson1 Practice Problems

Problem 1

Which square—large, medium, or small—covers more of the plane? Explain your reasoning.

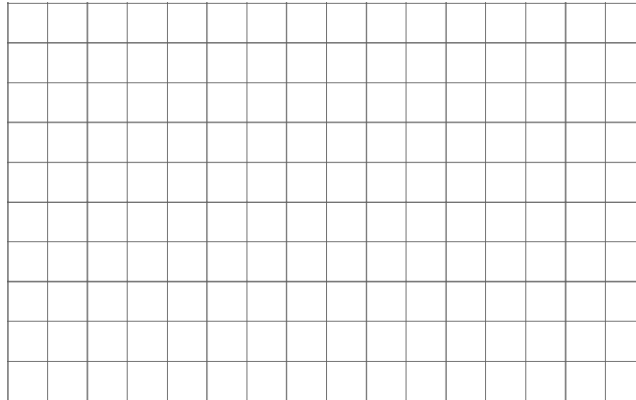


Possible Solutions

The large square covers more of the plane. Reasoning varies. Sample reasoning: A large square can fit exactly 9 small squares. A medium square can fit exactly 4 small squares. There are 5 large squares, which cover the same amount of the plane as 45 small squares. There are 10 medium squares, which cover the same amount of the plane as 40 small squares. There are only 10 small squares.

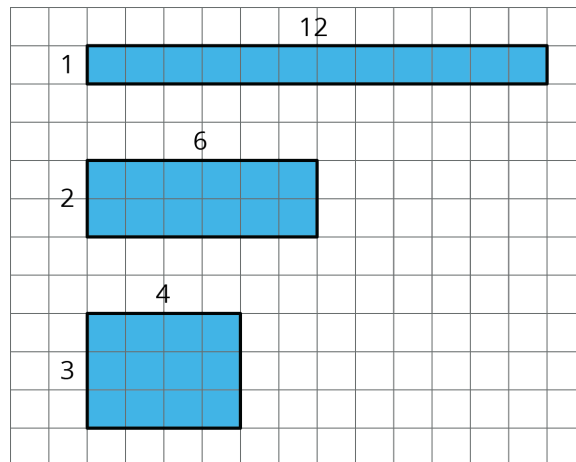
Problem 2

Draw three different quadrilaterals, each with an area of 12 square units.



Possible Solutions

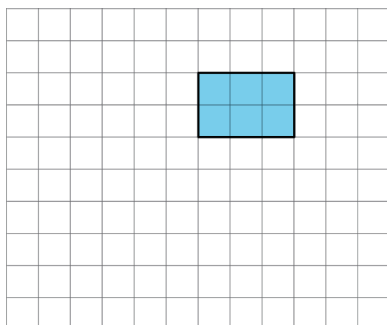
Answers vary. Sample response:



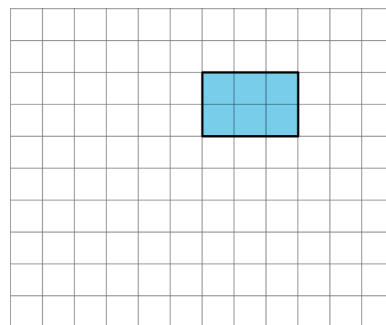
Problem 3

Use copies of the rectangle to show how a rectangle could:

a. tile the plane.

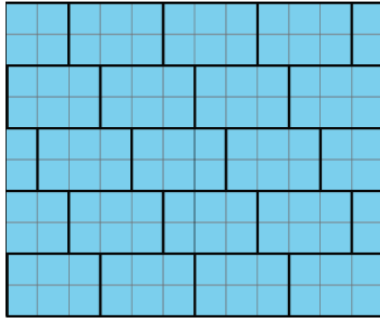


b. *not* tile the plane.

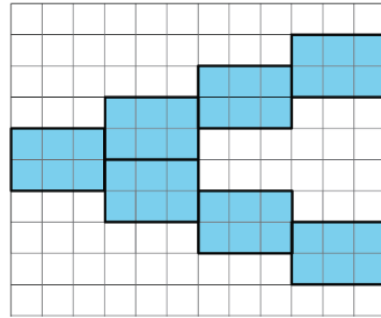


Possible Solutions

a. Answers vary. Sample response:

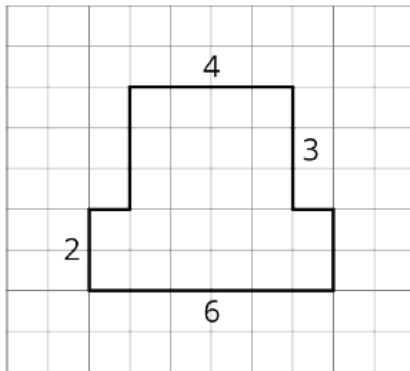


b. Answers vary. Sample response:



Problem 4

The area of this shape is 24 square units. Which of these statements is true about the area? Select all that apply.



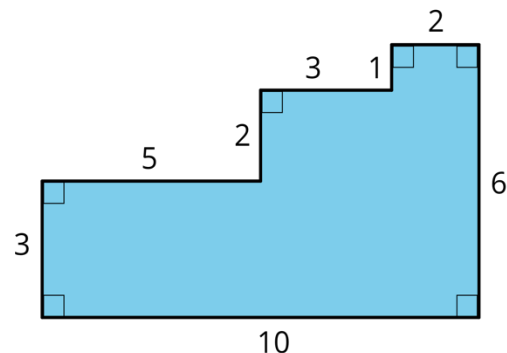
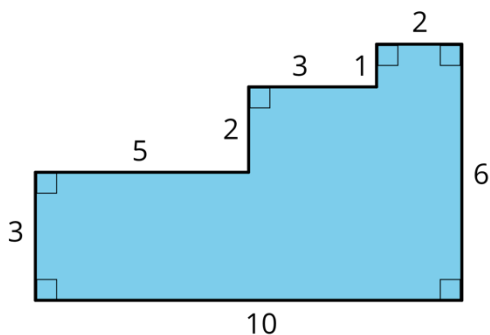
- A. The area can be found by counting the number of squares that touch the edge of the shape.
- B. It takes 24 grid squares to cover the shape without gaps and overlaps.
- C. The area can be found by multiplying the side lengths that are 6 units and 4 units.
- D. The area can be found by counting the grid squares inside the shape.
- E. The area can be found by adding 4×3 and 6×2 .

Possible Solutions

B, D, and E.

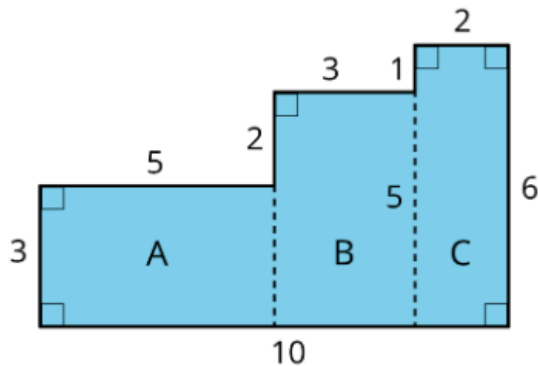
Problem 5

Here are two copies of the same figure. Show two different ways for finding the area of the shaded region. All angles are right angles.

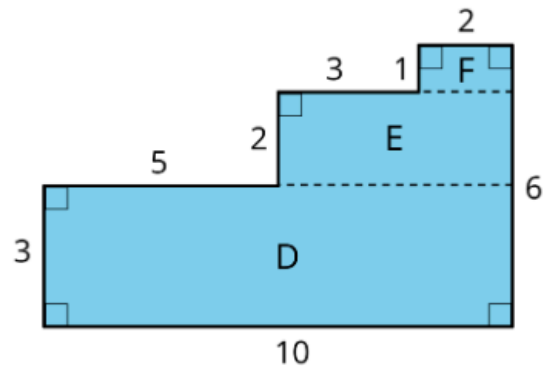


Possible Solutions

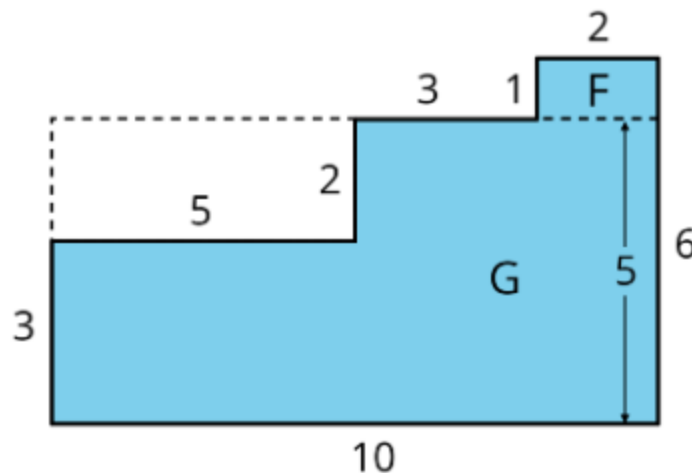
Answers vary. Sample strategies:



Area of A is 15 square units. Area of B is 15 square units. Area of C is 12 square units. The area of the entire region is $15 + 15 + 12$ or 42 square units.



Area of D is 30 square units. Area of E is 10 square units. Area of F is 2 square units. The area of the entire region is $30 + 10 + 2$ or 42 square units.



Area of F is 2 square units. Area of G is the area of the 10-by-5 rectangle subtracted by the area of a 5-by-2 rectangle in the upper left. $(10 \times 5) - (5 \times 2) = 50 - 10 = 40$, so the area of G is 40 square units. The total area is $40 + 2$ or 42 square units.

Problem 6

Which shape has a larger area: a rectangle that is 7 inches by $\frac{3}{4}$ inch, or a square with side length of $2\frac{1}{2}$ inches? Show your reasoning.

Possible Solutions

The square is larger. Its area is $2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2}$, which is $\frac{25}{4}$ or $6\frac{1}{4}$ square inches.

The rectangle has an area of $5\frac{1}{4}$ square inches because $7 \times \frac{3}{4} = \frac{21}{4}$.

Universal Design for Learners (UDL)

Supporting students with multiple means of...

Action and expression/modes of communication

5: Provide options for expression and communications

5.2 Use multiple tools for communication.

Students may use actual pattern blocks, paper cut-outs of the shapes, tracing paper, or an applet in Activity 1.1.

Representation

3: Provide options for comprehension

3.2: Highlight patterns, critical features, big ideas, and relationships

As students describe patterns, teacher listens to collect their ideas and then displays these for each Pattern (MLR2: Collect and Display)

Engagement

8: Provide options for sustaining effort and persistence

8.3 Foster collaboration and community

Students work individually and then share in small groups to determine a response; students work with partners to describe patterns.

Accommodations for Students with an Individual Education Program (IEP) or 504 Plan

Activity 1.2:

For students who benefit from hands-on activities, provide pattern tiles (or pre-cut an extra copy of each pattern) for students to compare the shapes.

For students with an Individual Education Program or a 504 Plan consult that student's individual document to plan and implement individual accommodations for this lesson.