What is the relationship between voltage applied to a DC motor and its RPM?

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Introduction .:. Top

Physicist Michael Faraday created the first electric motor in 1821 when he used copper wire and a horseshoe magnet to turn a wooden spindle ("Michael Faraday invented the first electric motor 200 years ago... and it's been pretty useful ever since!"), and in 1834, physicist Moritz von Jacobi would assemble the first usable direct current (DC) motor to drive a boat. This first real world application marked the beginning of the motor's imminent prevalence. Today, both alternating current (AC) and DC motors are used in all kinds of applications, with one notable example being the growing use in electric vehicles (Jeffries). An activity of mine that is relevant to this is my participation in the VEX Robotics Competition, where participants create robots that are primarily driven by battery-powered DC motors. As a result, I was interested in making them the subject of my investigation.

Fundamentally, DC motors convert electrical energy into mechanical energy through principles of electromagnetism (“Electric motor”). A stator, the stationary part of a motor, provides a magnetic field. A rotor, the rotary part of a motor, is a coil that is mounted to an axle. A commutator allows the rotor to spin continuously by reversing the current when half a revolution is made, and brushes connect the commutator to a power source. The power source provides a voltage (V) that causes electric current to pass through the system, as voltage is directly proportional to current according to Ohm’s law. The current produces a torque on the coil when it passes through the magnetic field (“Electric motor”). This all results in a spinning motor that may be of varying speed.

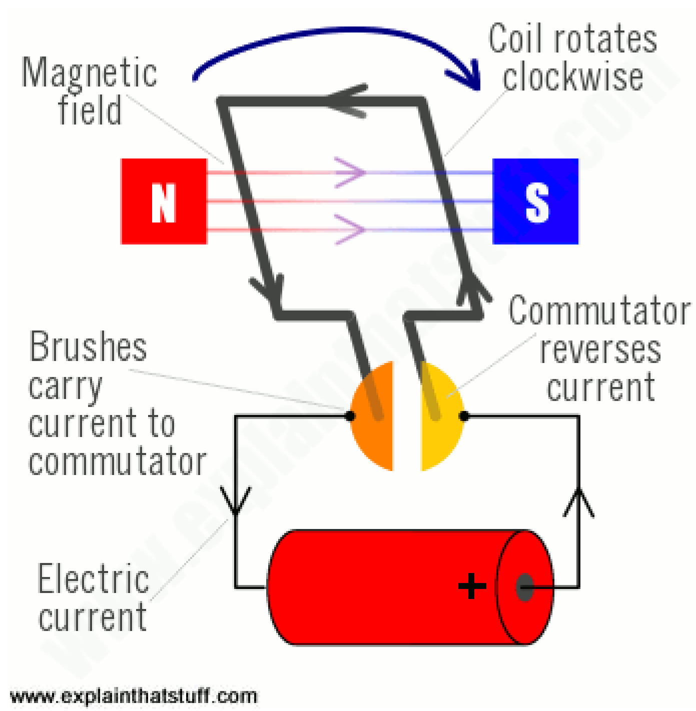

Figure 1: DC Motor Diagram (Woodford)

Therefore, this paper will investigate the research question: "What is the relationship between voltage applied to a DC motor and its RPM?". The independent variable is defined as the voltage applied to the motor. The dependent variable is defined as revolutions per minute (RPM).

I believe that the relationship between voltage applied and RPM of the motor will be graphically positive and linear. This is because, as mentioned before, the current (which produces torque) should be directly proportional to the voltage in compliance with Ohm's law.

Method .:. Top

In order to determine the relationship between voltage and angular velocity, I used the following setup. Fastened to a laboratory stand with a copious amount of adhesive tape was a sizable DC motor, with its shaft attached to a rotary motion sensor. A variable voltage power supply was connected to the positive and negative terminals of the motor, and a voltage probe bridged the two. The rotary motion sensor and voltage probe were each connected to a Vernier sensor interface that allowed a computer to collect data through Logger Pro software.

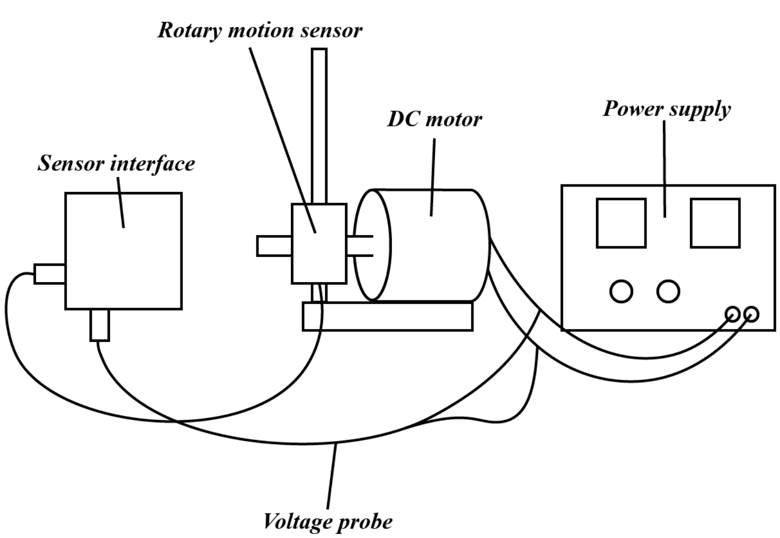

Figure 2: Procedure Setup Diagram

For the procedure, I decided that the data would be collected starting at 1.5 V, as this seemed to be the lowest voltage that the motor would run at. The interval chosen for each variation was an increase of 0.5 V as this was the smallest reasonable adjustment with the power supply, and I collected data from 1.5 V to a maximum of 6.0 V as any voltage applied beyond would abnormally output an angular velocity of 0 in the software. With each data point, I adjusted the voltage knob on the power supply until the computer software read the intended voltage, and set the potential in voltage, as well as the angular velocity in radians per second (rad/s) to be automatically collected for every tenth of a second within 5 seconds, therefore producing a sufficient 50 trials for each data point.

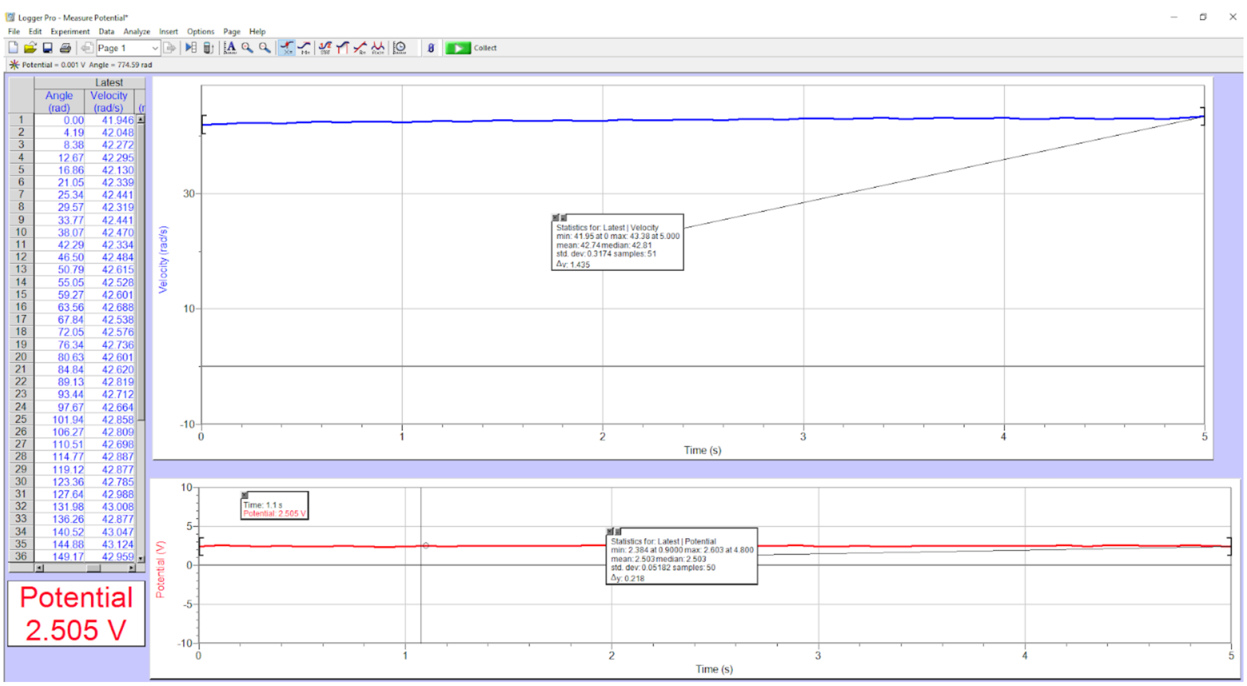

Figure 3: Example Computer Data Collection

To keep these variables from being influenced by other significant factors, such as vibration and temperature of the motor, I made sure that the motor was stably secured with the rotary motion sensor and not overheating before continuing with each variation. Additionally, the same power supply and motor were used for all trials to avoid any performance variation from different units or models.

There are no significant ethical or environmental concerns with this procedure, but one should keep in mind the common safety precautions of working with power supplies, such as using caution around the terminals while they are powered or avoiding the placement of liquids near the unit ("Safety Precautions of Power Supplies Cautions for Power Supplies"). One should also take care to avoid entanglement with the vigorous shaft of the motor.

Results .:. Data File .:. Top

Voltage Applied	RPM of Motor	
± 0.1	RPM	RPM
Voltage (V)	Average	Uncertainty
1.5	226.99	11.89
2.0	375.76	4.25
2.5	483.39	3.39
3.0	617.08	3.25
3.5	750.57	3.39
4.0	862.49	3.10
4.5	971.16	4.30
5.0	1153.56	5.25
5.5	1291.06	2.39
6.0	1420.94	1.91

Table 1: Processed Data Points Derived from Computer Data

The data table here displays the processed data points derived from collected computer data, as the hundreds of raw data points cannot be reasonably displayed. To average the data for each variation, the analyze function was used on Logger Pro. This automatically displayed the mean, as well as the minimum and maximum values of all trials within a 5 second domain (see figure 2). To calculate the uncertainty for one variation, half the range of the respective trials were taken. All values in rad/s (averages and uncertainties) were converted to RPM to address the research question in the same unit of measurement. The following is one sample of data processing:

The voltage applied to the motor was 3.0 V. After the five seconds of raw data was collected, I used the analyze function on Logger Pro, which yielded the output of 64.2 rad/s minimum velocity, 64.88 rad/s maximum velocity, and 64.62 rad/s mean velocity. I then calculated the uncertainty from the minimum and maximum values.

$$Uncertainty = \frac{max - min}{2}$$

$$Uncertainty_{3.0V} = \frac{64.88 - 64.20}{2}$$

$$Uncertainty_{3.0V} = 0.34\, rad/s$$

After attaining the average velocity and its uncertainty in rad/s, I converted each to RPM for the final result found in table 1.

$$RPM = \frac{rad}{s} \cdot \frac{60}{2\pi}$$

The average RPM at 3.0 V.

$$RPM_{average\ 3.0V} = 64.62\, rad/s \cdot \frac{60}{2\pi}$$

$$RPM_{average\ 3.0V} \approx 617.08\ RPM\ (5\ s.f.)$$

The uncertainty of the RPM at 3.0 V.

$$RPM_{uncertainty\ 3.0V} = 0.34\ rad/s \cdot \frac{60}{2\pi}$$

$$RPM_{uncertainty\ 3.0V} \approx 3.25\ RPM\ (3\ s.f.)$$

Additionally, the analyze function on Logger Pro was applied to the voltage readings. This yielded the aforementioned values (mean, minimum, maximum) but for the voltage applied, which allowed for a similar calculation of the uncertainties for each variation. The average of these uncertainties was taken to obtain a general uncertainty of 0.1 V for the independent variable, assuming that it is primarily the result of random error (such as in analog adjustment).

I then compiled the data from table 1 into the following graph using Google Sheets.

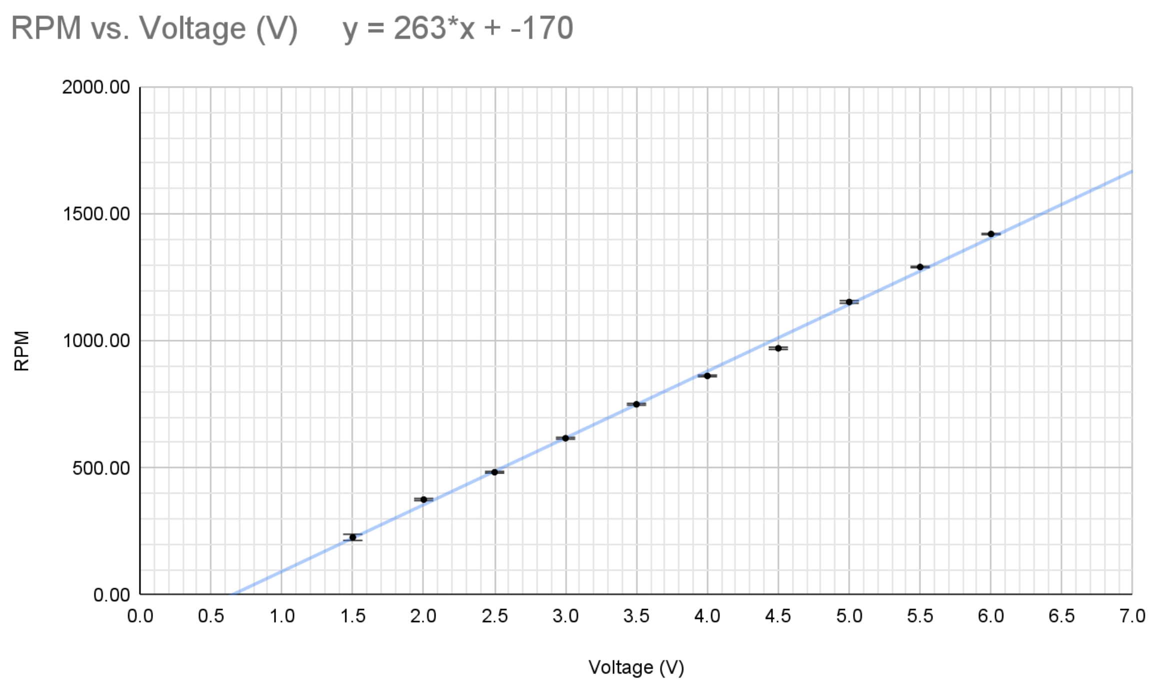

Figure 4: Relationship between Voltage and RPM with Error Bars

Calculating the slope uncertainty of the trendline using the maximum and minimum slopes through the error bars was not possible by hand due to their negligible size. Using the LINEST function in Google Sheets instead, the uncertainty of the slope was calculated to be 4.29 RPM/V. Therefore, the slope of the trendline is 263 ± 4.29.

Interpreting this graph, it can be visually seen that the relationship between voltage and RPM is positive and linear. Also, the relatively minimal slope uncertainty of the trendline suggests that the linear trendline is a good fit for the data. Another interesting note is the non-zero y-intercept of the trendline. This means that (at least for this particular motor) a certain voltage must be reached for any RPM to be produced.

Conclusion .:. Top

Considering the results of the graph, as well as the small uncertainty of the slope, it can be confidently deduced that the relationship between voltage applied to a DC motor and its RPM is positively linear. It also appears that this conclusion may support my original hypothesis that the relationship would be graphically positive and linear due to Ohm's law. Since current is what generates a torque on the coil of the motor, and Ohm's law states that voltage is directly proportional to current, it would make sense that voltage would then increase with RPM.

There were some limitations to this investigation. One was with the power supply used, because the voltage applied had to be manually adjusted with a knob. Although a digital reading was measured, the physical knob lacked the precision to get the exact intended voltage, which resulted in some uncertainty with the independent variable. Another issue was the way in which the motor was adhered to the laboratory stand. It seemed fairly stable for my data collection, but tape is not a completely rigid or consistent solution. The motor could be more susceptible to misalignment with the rotary motion sensor or vibration if not secured properly. In addition, the rotary motion sensor did not output a result past 6.0 V applied, which limited the amount of variations that could have been collected. Finally, the graph of the relationship (figure 3) displayed a y-intercept of less than zero, which could indicate a possible zero error if unable to be ascertained. This may affect the accuracy of this investigation.

Of course, there are some realistic and relevant suggestions for improvement. The issue with the imprecise voltage could be solved by use of a digital power supply with fine adjustment. This would allow for a greater accuracy in the voltage applied and therefore reduce the uncertainty of the independent variable. For the motor mounting, some kind of stable clamp, ideally with dampening, could improve stability of the motor and reduce any outside effects from vibration or misalignment with the rotary motion sensor. A different model rotary motion sensor could also be used, which may allow a higher number of variations to be tested that may or may not support the result found here.

One point for future research could be the aforementioned non-zero y-intercept. If not a zero error in measurement, the cause for the lack of movement before ≈ 0.7 V could be investigated. It may be due to friction, cogging torque, or some other force that causes an initial stiffness in the motor. Investigating this may provide an explanation for why the relationship found here is only positively linear, and not directly proportional. Something else that could be further explored is the effect of temperature on the motor's RPM. Temperature has the capability to affect resistance, so it would be interesting to see if it has any effect on the RPM of a motor as the conducting material of the motor becomes less ohmic. Lastly, calling back to my participation in the VEX Robotics Competition, I know that the application of motors there typically involves significant loads. So, I would be curious to see future research on the relationship between voltage and torque generated, and maybe I could apply what I have found to my own work.

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Related Links .:. Top

- https://en.wikipedia.org/wiki/DC_motor - Wikipedia - Some more information on the same kind of motor I used
- https://en.wikipedia.org/wiki/Ohm%27s_law - Wikipedia - Stuff about Ohm’s law which could help explain why the relationship was not directly proportional
- https://www.ncbi.nlm.nih.gov/pmc/articles/PMC8752902/ - dos Reis et al. - A research paper further exploring motor characteristics with things like load and gearing
- https://www.matec-conferences.org/articles/matecconf/pdf/2018/84/matecconf_ses2018_03010.pdf - Abdullah et al. - A research paper on the temperature rise of a motor under different loads

- https://ophysics.com/em10.html - Fun DC motor simulation with settings for magnetic field, loops and voltage