Name:	Class: Sec _____	Index No.: _____

4E

MERIDIAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020

ADDITIONAL MATHEMATICS **Paper 1**

4047/01 31 August 2020

SECONDARY 4 EXPRESS **2 hours**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The total marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

For Examiner's Use	
Total Marks	

This question paper consists of **16** printed pages and **2** blank pages.

Setter: Mdm Shermane Yeo **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The curve $y = px^q - 15$ passes through the points (2, 195) and (3, 125). Calculate the value of p and of q. [4]

2 **(i)** Show that $(x - 1)$ is a factor of $\mathrm{f}(x) = x^3 - (2p+1)x^2 + (2p-q)x + q$ where p and q are constants. [1]

(ii) Explain, with working, why the equation f $(x) = 0$ will have 3 real roots if $p^2 + q \geq 0$. [4]

3 **(i)** Sketch the graph of $y = -|2x - 5|$. [2]

A line has the equation $y = mx - 2$.

(ii) Find the value of m if the line meets the graph of $y = -|2x - 5|$ at $x = 3$. [2]

(iii) Determine the range of values of m for which the equation $mx - 2 = -|2x - 5|$ has no solution. Support your answer with clear reasoning. [2]

4 **(i)** Prove that $\tan A\left(\text{cosec}2A + \cot 2A\right) = 1$. [3]

(ii) Hence find all the angles of A between 0 and 2π which satisfy the equation $\tan A\left(\text{cosec}2A + \cot 2A\right) + \sec 2A = 4$. [3]

5 **(a)** The magnitude of an earthquake is measured by the Richter scale. The magnitude, M, can be determined by the equation $M = \lg\left(\frac{I}{I_0}\right)$, where I is the intensity of seismic waves recorded by seismographs and I_0 is a constant which represents the threshold intensity.

(i) State the magnitude of an earthquake such that its intensity is the same as the threshold intensity. [1]

(ii) Earthquakes of magnitude 5.8 and 6.3 occurred in Taiwan and Canada respectively in 2019. Find the value of $\frac{I_C}{I_T}$ as a power of 10, given that I_C and I_T represent the intensity of the earthquakes in Canada and Taiwan respectively. [3]

(b) Solve the simultaneous equations

$$2^{p-9} \div 8^{q} = \sqrt[4]{32^{p}}$$

$$\log_2 6 - \log_4 (11q - 2p) = 1$$

[5]

6 A curve is given by the equation $y = x^2\sqrt{8+kx}$.

(i) Show that the value of k is -2 given that the curve has a turning point at $x = 3.2$.

[4]

(ii) Using the value of k found in part **(i)**, find the range of values of x for which the curve is decreasing. [3]

7 The function f is defined, for $-2\pi \le x \le 2\pi$, by $\mathrm{f}(x) = -3\cos\frac{x}{2} + 1$.

(i) State the amplitude of $\mathrm{f}(x)$. [1]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $-2\pi \le x \le 2\pi$. [2]

(iii) On the diagram drawn in part **(ii)**, sketch the graph of $y = \frac{x}{\pi} - 1$ for $-2\pi \le x \le 2\pi$. [1]

(iv) Hence state the number of solutions of the equation $3\cos\frac{x}{2} + \frac{x}{\pi} = 2$ for $-2\pi \le x \le 2\pi$. [1]

8 It is given that $\frac{d^2y}{dx^2} = \frac{1}{2x^{\frac{3}{2}}}$. The curve intersects the x-axis at P and the equation of the tangent to the curve at point P is $y = \frac{1}{2}x - 2$. Find the equation of the curve. [6]

9 It is given that $\tan\alpha = p$ and $\tan(\alpha - \beta) = q$.

(i) Express $\tan\beta$ in terms of p and q. [3]

(ii) Calculate the value of $\tan(\alpha + \beta)$ when $p = 1$ and $q = 0.5$. [3]

10 The roots of the equation $x^2 = x + 7$ are α and β.

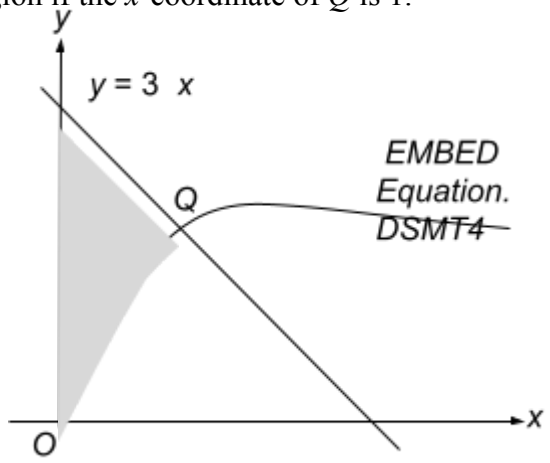
(i) Prove that $\frac{1}{\alpha} = \frac{\alpha - 1}{7}$. [2]

(ii) Write down a similar expression for $\frac{1}{\beta}$. [1]

(iii) Hence or otherwise, find a quadratic equation whose roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$. [4]

11 **(a)** Given that $y = \dfrac{\ln\left(e^x - x\right)}{2}$, find $\dfrac{dy}{dx}$. [2]

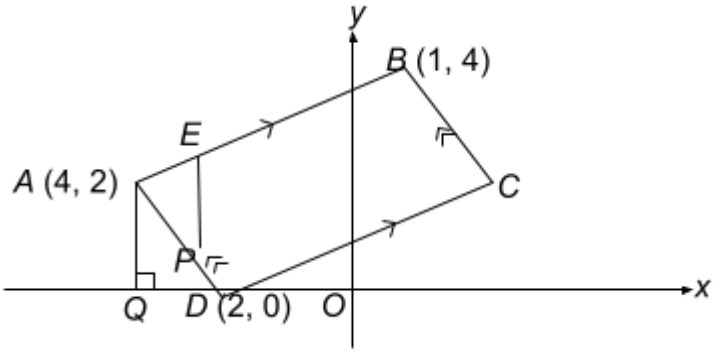
(b) In the diagram below, Q is the point of intersection of the curve $y = \dfrac{2e^x - 2}{e^x - x}$ and the line $y = 3 - x$. Using the result found in part **(a)**, find the area of the shaded region if the x-coordinate of Q is 1. [6]



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12 Solutions to this question by accurate drawing will not be accepted.

ABCD is a parallelogram where *A* is (−4, 2), *B* is (1, 4) and *D* is (−2, 0). The point *E* lies on *AB* such that $AE : EB = 2 : 5$. Lines are drawn, parallel to the *y*-axis, from *A* to meet the *x*-axis at *Q* and from *E* to meet *AD* at *P*.



(i) Calculate the coordinates of *C* and of *E*. [4]

(ii) Find the equation of AD and calculate the coordinates of P. [3]

(iii) Explain why $AEPQ$ is a parallelogram and calculate its area. [4]

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