Exam 3 Review

Sec 5.6, 5.7, 7.1, 7.2, 7.3, 7.4

1. Use the method of reduction of order to find the general solution to $x^2y'' - xy' + y = x$ given that $y_1 = x$ is a solution to the complementary equation.
2. Use the method of reduction of order to find the general solution to $xy'' - (2x + 2)y' + (x + 2)y = 0$ given that $y_1 = e^x$ is a solution.
3. Find the general solution: $x^2y'' - xy' + y = 0$.
4. Find the particular solution to $3x^2y'' - xy' + y = 0$ satisfying $y(8) = 26,\ y'(8) = \frac{37}{12}$.
5. Given the differential equation $y'' - xy' - y = 0$:
   a. Suppose that $y(x)$ has a Taylor series about $x = 0$,
   $y(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + ...$
   Substitute into the differential equation and simplify by grouping together terms with similar powers of *x*.
   b. Given the initial conditions$y(0) = 16,\ y'(0) = 15$, find the first five terms of the Taylor series solution $y(x)$.
   c. Use the answer to part b to find an approximation of $y(2)$
6. Given the differential equation $y'' + x^2y = 0$ with initial conditions$y(0) = 1,\ y'(0) = 0$, use the first five terms of the Taylor series about $x = 0$ to find an approximate value of the solution at $x = 1.2$ (*HINT: Follow the same steps as in the previous problem).*
7. Find a particular solution of $x^2y'' + xy' - y = 2x^2 + 2$ given that $y_1 = x$ and $y_2 = \frac{1}{x}$ are solutions of the complementary equation.
8. Find a particular solution of $xy'' + (2 - 2x)y' + (x - 2)y = e^{2x}$ given that $y_1 = e^x$ and $y_2 = \frac{e^x}{x}$ are solutions of the complementary equation.
9. Find the general solution: $y'' - 2y' + y = 14x^{3/2}e^x$

Exam 3 Review ANSWER KEY

If you discover an error please let me know, either in class, on the OpenLab, or by email to jreitz@citytech.cuny.edu. Corrections will be posted on the OpenLab.

1. $y = c_1 x + c_2 x \ln(x) + \frac{1}{2} x[\ln(x)]^2$
2. $y = c_1 e^x + c_2 e^x x^3$
3. $y = c_1 x + c_2 x \ln(x)$
4. $y = 3x + x^{1/3}$
5. a.
   $(2a_2 - a_0) + (6a_3 - 2a_1)x + (12a_4 - 3a_2)x^2 + (20a_5 - 4a_3)x^3 + (30a_6 - 5a_4)x^4 + ... = 0$
   b. Taylor series at $x = 0$ given initial conditions $y(0) = 16,\ y'(0) = 15$:
   $y(x) \approx 16 + 15x + 8x^2 + 5x^3 + 2x^4 + ...$
   c. $y(2) \approx 150$
6. Taylor series at $x = 0$: $y(x) \approx 1 - \frac{1}{12}x^4$. Approximate value at $x = 1.2$is $y(1.2) \approx 0.8272$
7. $y_p = \frac{2}{3}(x^2 - 3)$
8. $y_p = \frac{e^{2x}}{x}$
9. *HINT: First solve the complementary equation. Then use variation of parameters.*
   $y = \frac{8}{5}x^{7/2}e^x + c_1 e^x + c_2 x e^x$