ESCUELA POLITÉCNICA NACIONAL
MATEMÁTICA AVANZADA
CORRECCION PRUEBA 1

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TEMA: CORRECCION PRUEBA 1
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CORRECCION DE LA PRUEBA

1.- Sea $\omega = \frac{1+z}{1-z}$; donde $z = \cos\alpha + i\,\text{sen}\,\alpha$. Hallar ω.

$$\omega = \frac{1+z}{1-z} = \frac{1+\cos\alpha + i\,\text{sen}\,\alpha}{1-(\cos\alpha + i\,\text{sen}\,\alpha)}$$

$$= \frac{1+\cos\alpha + i\,\text{sen}\,\alpha}{(1-\cos\alpha) - (i\,\text{sen}\,\alpha)} \cdot \frac{1-\cos\alpha + i\,\text{sen}\,\alpha}{1-\cos\alpha + i\,\text{sen}\,\alpha}$$

$$= \frac{[(1+i\,\text{sen}\,\alpha) + \cos\alpha]\,[(1+i\,\text{sen}\,\alpha) - \cos\alpha]}{(1-\cos\alpha)^2 + \text{sen}^2\alpha}$$

$$= \frac{(1+i\,\text{sen}\,\alpha)^2 - \cos^2\alpha}{1-2\cos\alpha + \cos^2\alpha + \text{sen}^2\alpha}$$

$$= \frac{1+2i\,\text{sen}\,\alpha - \text{sen}^2\alpha - \cos^2\alpha}{1-2\cos\alpha + 1}$$

$$= \frac{\cancel{1} + 2i\,\text{sen}\,\alpha - \cancel{1}}{2-2\cos\alpha} = \frac{\cancel{2}i\,\text{sen}\,\alpha}{\cancel{2}(1-\cos\alpha)}$$

$$= i\,\frac{\text{sen}\,\alpha}{1-\cos\alpha}$$

2.- Resolver

a.- $e^z = \frac{1+3i}{2}$

$$e^z = e^{x+iy}$$
$$= e^x e^{iy}$$
$$= e^x(\cos y + i\,\text{sen}\,y) = \frac{1}{2} + \frac{3}{2}i$$

$e^x \cos y = \frac{1}{2}$ $\qquad$ $e^x\,\text{sen}\,y = \frac{3}{2}$

$e^x = \frac{1}{2\cos y}$ $\qquad$ $\frac{1\,\text{sen}\,y}{2\cos y} = \frac{3}{2}$

$\qquad$ $\tan y = 3$

$e^x = \frac{1}{2\cos 71{,}56}$ $\qquad$ $y = 71{,}56$

$e^x = 1{,}58$

$\ln e^x = \ln 1{,}58$

$x = 0{,}46$

$$e^z = e^{0{,}46}(\cos 71{,}56 + i\,\text{sen}\,71{,}56)$$
$$e^z = 0{,}5 + i\,1{,}5$$

b- $\omega = (\sqrt{2} - i)^{1-i}$

$z_1 = \sqrt{2} - i$

$z_2 = 1 - i$

$$\omega = e^{(1-i)\ln(\sqrt{2} - i)}$$

$|z_1| = \sqrt{3}$

$\Theta = \arctan\left(\frac{-1}{\sqrt{2}}\right) = -35,26$
$= 324,73$

$$\omega = e^{(1-i)(\ln\sqrt{3} + i(5,66 + 2\pi k)}$$

$$\omega = e^{(1-i)(\ln\sqrt{3} + 5,66 i)}$$

$$\omega = e^{(1-i)(0,55 + 5,66 i)}$$

$$\omega = e^{0,55 + 5,66 i - 0,55 i + 5,66}$$

$$\omega = e^{6,21 + 5,11 i}$$

3- Sea $f(z) = \frac{z\,Re(z^4)}{|z|^2} \cdot z \neq 0$

¿Como podemos definir $f(z)$, tal que f sea continuo, $\forall z \in \mathbb{C}$?

i) $f(0)$ = no existe o no esta definido

ii) $\exists \lim_{z \to 0} f(z)$

$z = x + iy$

$z^4 = (x^2 + 2xiy + y^2)(x^2 + 2xiy + y^2)$

$z^4 = x^4 + i2x^3y - x^2y^2 + i2x^3y - 4x^2y^2 - i2y^3 - x^2y^2 - i2xy^3 + y^4$

$Re(z^4) = x^2 - 6x^2y^2 + y^4$

$|z| = \sqrt{x^2+y^2}$

$|z|^2 = x^2 + y^2$

$$\lim_{(x,y) \to 0} \frac{(x+yi)(x^2 - 6x^2y^2 + y^2)}{(x^2 + y^2)}$$

$$\lim_{(x,y) \to (0,0)} \frac{x^3 - 6x^3y^2 + xy^2 + x^2yi - 6x^2y^3i + y^3 i}{x^2+y^2}$$

• Si $y = kx$

$$\lim_{x \to 0} \frac{x^3 - 6x^3 \cdot k^2x^2 + xk^2x^2 + x^2kxi - 6x^2k^3x^3 i + k^3x^3 i}{x^2 + k^2x^2}$$

$$\lim_{x \to 0} \frac{x^3 - 6x^5k^2 + k^2x^3 + x^3ki - 6x^5k^3 i + k^3x^3 i}{x^2(1+k^2)}$$

$$\lim_{x \to 0} \frac{x^2(x - 6x^3k^2 + k^2x + kxi - 6x^3k^3 i + k^3xi)}{x^2(1+k^2)} = 0$$

$$\lim_{z \to 0} \frac{z\,Re(z^4)}{|z|^2} = 0 \rightarrow \forall \varepsilon > 0 \quad \exists \delta > 0$$

$\left| \frac{z\,Re(z^4)}{|z^2|} \right| < \varepsilon \rightarrow 0 < |z| < \delta$

$\frac{|z\,Re(z^4)|}{|z|^2} < \varepsilon.$

$|Re(z^4)| < |z|^4$

$\frac{|z|\,|z|^4}{|z|^2} < \varepsilon$

$|z|\,|z|^2 < \varepsilon$

$|z|^2 < \varepsilon$

$|z| < \sqrt{\varepsilon}$

$\delta = \sqrt{\varepsilon}$

$\therefore \lim_{z \to 0} \frac{z\,Re(z^4)}{|z|^2} = 0$

Redefiniendo

$$f(z) \begin{cases} \frac{z\,Re(z^4)}{|z|^2} & ; \text{si } z \neq 0 \\ 0 & \text{si } z = 0 \end{cases}$$

4.- Utilice la definición de derivada para evaluar $f'(z_0)$, o pruebe que:

$f'(z)$ no existe, si $f(z) = \dfrac{z}{1+z}$ $\quad z_0 = 2$

$$\exists \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$\lim_{z \to 2} \frac{\frac{1}{1+z} - \frac{1}{1+2}}{z-2}$$

$$\lim_{z \to 2} \frac{\frac{3-1-z}{3(1+z)}}{z-2}$$

$$\lim_{z \to 2} \frac{2-z}{3(z-2)(1+z)}$$

$$\lim_{z \to 2} \frac{2-z}{3(z-2)(1+z}$$

$$\lim_{z \to 2} \frac{2-z}{3(z-2)(1+z)}$$

$$\lim_{z \to 2} \frac{-(z-2)}{3(z-2)(1+z)}$$

$$\lim_{z \to 2} \frac{-1}{3(1+z)}$$

$$\lim_{z \to 2} \frac{-1}{3(1+2} = -\frac{1}{9}$$ $\therefore$ $f(z)$ si es derivable en $z_0 = 2$