

## Lesson Plan Assignment

### Overview

**Lesson planners (your names):** Zeinab Abdullah and Azizi Jarouche

**Grade level:** 4th

**Source of Original Lesson:**

Houghton Mifflin Company (2007). Houghton Mifflin Math. Grade 4. Boston, MA: Houghton Mifflin Company.

**Brief description of original lesson:**

**Critique of original lesson:** (Look closely at how the lesson is presented. What are strengths? What needs improvement?)

We think that a strength of this lesson is it allows students to identify fractions in different ways. For example, in this lesson students are identifying fractions by looking at pictures and then they are making their own drawings based on given fractions. Another strength is that the lesson identifies the different parts of a fraction (numerator and denominator) and explains how fractions are supposed to be read. A common mistake students make when reading a fraction is that they say 1 over 4, but this lesson makes it clear that we read it as "one fourth." A third strength is during guided practice the lesson has an "Ask yourself" section which allows the students to think and reason about what the different parts of the fraction represent. An area that needs improvement is the lack of opportunities for students to work together and learn from each other's reasonings. Students are not given the opportunity to work in small groups or in pairs to solve the different problems. The lesson also does not allow for hands-on activities, and requires more writing and drawings than doing. For example, the lesson could have students create fraction strips and locate the fractions on the strips instead of just drawing a picture to represent the fractions. In the textbook lesson, questions 1-7 give students specific drawings to represent the different fractions. Students might just imitate that method of drawing fractions when asked to draw their own. If the lesson had students create fraction strips, number lines, diagrams, or create fraction cards instead; the students would have had to rely more on their background knowledge and their understanding of the lesson to do it, than just use a model that is provided in a previous question for them. The questions in this lesson limit the models students have to use. All the questions only require drawings of pictures, none asked for number lines or comparing fractions to a whole.

**Lesson Plan****Template (based on Smith et al. 2008)****Lesson Title/Topic:** Ordering Fractions 2.2/Fraction Cards**Grade Level:** 4**Number of Class Periods:** 1**Text or Resource:**

TERC. (2008). Investigations in number, data, and space (2nd ed.). Grade 4, Fraction Cards and Decimal Squares. Glenview, IL: Pearson.

(Include unit, session, and pages)

TERC Gr4 U6 Ordering Fractions 2.2, pages 74-77

**Overview:** (State briefly what happens in your lesson)

In the Fraction Cards lesson, students will work with their small groups of 4 students to make fraction cards. Each student in the group will make 10 fraction cards, so each small group will have a total of 40 fraction cards. Before students start making their fraction cards the teacher will show students fraction cards she previously made to represent  $\frac{3}{4}$  and  $1\frac{1}{2}$ . Students will refer to their *Student Activity Book* page 27 to make their fraction cards. Before students start working on the task, the teacher will read and explain the task in detail by going over the directions. Page 27 on the student's activity book has four columns with 10 fractions in each column; each student in the group will be responsible for creating fraction cards for one column. Students will be given blank wholes, blank thirds and blank fifths to create their fraction cards. Students will be advised to use the blank thirds to help them create the sixths and twelfths, and the fifth to help them create the tenths. Students will shade in the part of the whole that represents the numerator. For example, for  $\frac{3}{4}$  students will shade 3 parts of the 4 parts.

After making the Fraction Cards, students will work on their *Student Activity Book* page 31. On this page, students will be given different Fraction Cards. Students will be asked to write the name of the shaded part of each of those fraction cards. Then, students will draw an equivalent fraction to one of those fraction cards, and write an explanation as to why they think those fractions are equivalent. Throughout the lesson, students will continue to work on creating different fraction cards to represent different fractions that could range from  $0/2$  to  $2\frac{1}{2}$ . In addition to working with creating fraction cards, students will work on an activity called "One Piece is Missing". Students will determine what "one piece missing means." They will look at two fractions that are one part away from the whole and determine whether they are equivalent or one is larger than the other. They will use drawings and examples to support their arguments.

**Learning Goals/ Outcomes:**

- **Content specific to this lesson**

*Math content in this lesson:*

- Students will interpret what the numerator and denominator of a fraction mean.
- Students will learn about the meaning of "one piece is missing."
- Students will be able to represent different fractions using Fraction Cards.
- Students will "compare fractions that are one fractional part less than 1" (p. 77).

- Students will compare two fractions to determine if they are equivalent or if one is greater than the other.

**PSSM:** (copy and paste from Number and Operations Standards)

- “Understanding numbers, ways of representing numbers, relationships among numbers, and number systems” (p. 148)
  - Develop understanding of fractions as parts of unit wholes, as parts of a collection, as location on number lines, and as division of whole numbers;
  - Use models, benchmarks and equivalent forms to judge the size of fractions.

**CCSS:** (copy and paste from the CCSS document)

- **Extend understanding of fraction equivalence and ordering.**
  - CCSS.MATH.CONTENT.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

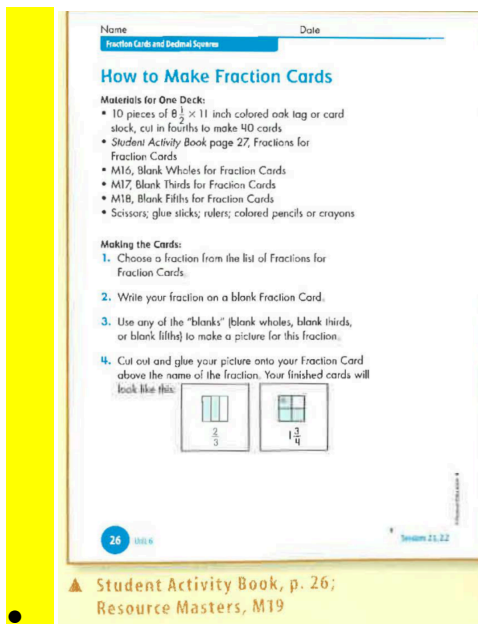
• **PSSM Process Standards:**(using 5 process standards)

- **Representation:** Students will use the different fraction cards they made to represent different fractions.
- **Communication:** Students will be asked to share and discuss their answers with their elbow partners before sharing with the whole class.
- Students will be asked to share their solutions and reasonings and discuss with the class during whole class discussions.
- Students will be asked to work in small groups and will discuss with their small groups how to create the different fraction cards.
- **Reasoning and Proof:** Students will come up with arguments for whether two fractions are the same size. Students will use drawings and examples to support their reasoning and argument. Students will use reasoning to “compare fractions that are one fractional part less than one” (p. 77).
- **On-going goals:** beyond the math content and processes, what are other social or affective goals you are working on throughout the year?
  - Students will work with their peers to discuss and learn from each other’s reasonings.
  - Students will be open to respectfully listen to each other’s reasonings, and question any misunderstandings.
  - Students will be willing to share their solutions regardless if right or wrong.
  - The class environment will be appropriate for students to feel comfortable and safe to share and discuss their errors.
  - Students will provide each other with constructive and respectful feedback to support their classmate’s learning.

**Materials/tools/resources:** Specify materials for teacher to prepare in advance and materials children will use.

- Student Activity Book, pp. 26-27, 31
- M16 (as needed); M17 (as needed); M18 (as needed)
- Colored oak tag or card stock; scissors;
- glue sticks; rulers; colored pencils or crayons
- Student Activity Book, p.32
- Student Math Handbook, pp. 56-57

Source: 2.2 (p. 74)



**Use of Space/grouping:** What parts of the lesson use individuals, pairs, small groups, full class?

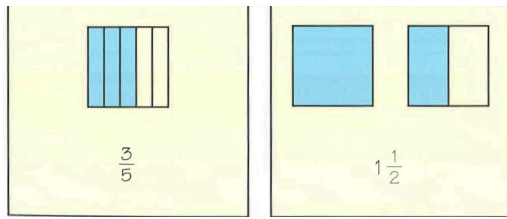
- According to 2.2 TERC, the Making Fraction Cards activity will be done in groups. The One Piece is Missing activity will be done as a whole class and in pairs. The daily practice will be done individually.

**Time estimates:** How long do you expect to spend on each portion of your lesson?

- Making Fraction Cards (40 minutes, Groups)
- One piece is missing (20 minutes, Class/pairs)
- Daily Practice (Time may vary according to the student's proficiency)

**Relation to students' previous knowledge:** (What has come before that they will use?)

In the previous lesson (2.1), students learned about comparing and ordering fractions. They decided whether adding fractions such as  $\frac{1}{2} + \frac{3}{6} + \frac{4}{8}$  will give them a sum greater or less than one. Students also learned how to represent fractions using Fraction Cards and 4 x 6 rectangles. They learned how to represent fractions that are less than or greater than one. For example, they represented  $\frac{2}{3}$  and  $1 \frac{3}{4}$  like the following:



(Source: 2.1, pg. 71)

This will help students in this lesson because they will continue to use fraction cards and 4 x 6 rectangles to represent fractions. In the previous lesson students learned about mixed fractions, so they had opportunities to make connections between fractions and whole numbers, and understand that fractions do not necessarily have to be between 0 and 1. This will help students in this lesson because students are comparing fractions that are one part away from the whole and will need to think about the distance each fraction is away from the whole. Students will be able to connect what they learned in the previous lesson to help them realize that the closer a fraction is to one or the whole, the larger the fraction. Students have also learned about what the numerator and denominator represent. For example, when given the fraction  $\frac{3}{5}$  students know that the numerator 3 represents 3 parts of the whole and the denominator 5 represents the whole. This will help students when comparing two fractions.

**Task for the Lesson:** Brief description of the mathematically worthwhile task you will use (e.g., the students will represent different combinations of five crayons using two colors.) Copy and paste worksheet, activity directions, handouts, or game directions.

Students will create Fractions Cards. The directions and materials for the Fraction Cards are below. Then they will use the fraction cards they have created and will look at two fractions that are one part away from the whole such as  $\frac{3}{4}$  and  $\frac{5}{6}$ . They will determine and come up with an argument whether they are the same size or whether one is larger than the other. Students will be encouraged to “use drawings and further examples of other fractions that have ‘piece missing’ (e.g.,  $\frac{7}{8}$ ,  $\frac{2}{3}$ ) in their argument” (p. 77).

Name \_\_\_\_\_ Date \_\_\_\_\_

**Fraction Cards and Decimal Systems**

### How to Make Fraction Cards

**Materials for One Deck:**

- 10 pieces of  $8\frac{1}{2} \times 11$  inch colored oak tag or card stock, cut in fourths to make 40 cards
- Student Activity Book page 27, Fractions for Fraction Cards
- M16, Blank Wholes for Fraction Cards
- M17, Blank Thirds for Fraction Cards
- M18, Blank Fifths for Fraction Cards
- Scissors; glue sticks; rulers; colored pencils or crayons

**Making the Cards:**

1. Choose a fraction from the list of Fractions for Fraction Cards.
2. Write your fraction on a blank Fraction Card.
3. Use any of the “blanks” (blank wholes, blank thirds, or blank fifths) to make a picture for this fraction.
4. Cut out and glue your picture onto your Fraction Card above the name of the fraction. Your finished cards will look like this:

26

Student Activity Book, p. 26;  
Resource Masters, M19

(Source 2.1, pg. 71)

Students will follow the directions above to create their own fraction cards which they will use to compare fractions.

**Teaching Note**

➤ **Identifying Each Group's Fraction Cards** Each group needs to be able to recognize their deck of Fraction Cards. If you do not have enough different colors of oak tag for all the decks, each group should mark the backs of all the cards in their deck with an identifying mark or colored scribble to differentiate them from the other decks in the class.

Student Activity Book, p. 27

(Source 2.1, pg. 72)

Students will use these fractions to make their fraction cards.

**Launch** (introduce and engage students with the task without diminishing cognitive demand; verify students understand the task)

We will start the lesson by explaining to students that they will continue to work on their Fraction Cards that they started on in the previous lesson.

We will tell students “When you are done making your own fraction cards, you will work with your group to make cards for the class deck. Mark the class deck with the color blue so you don’t mix them with your own cards. Here are the directions to the Fraction Cards. Use fractions that are listed on page 27.”

**How to Make Fraction Cards**

**Materials for One Deck:**

- 10 pieces of 9 1/2 x 11 inch colored oak tag or card stock, cut in fourths to make 40 cards
- Student Activity Book page 27, Fractions for Fraction Cards
- M16, Blank Wholes for Fraction Cards
- M17, Blank Thirds for Fraction Cards
- M18, Blank Fifths for Fraction Cards
- Scissors; glue sticks; rulers; colored pencils or crayons

**Making the Cards:**

1. Choose a fraction from the list of Fractions for Fraction Cards.
2. Write your fraction on a blank Fraction Card.
3. Use any of the “blanks” (blank wholes, blank thirds, or blank fifths) to make a picture for this fraction.
4. Cut out and glue your picture onto your Fraction Card above the name of the fraction. Your finished cards will look like this:

Student Activity Book, p. 26; Resource Masters, M19

**Teaching Note**

➤ **Identifying Each Group's Fraction Cards** Each group needs to be able to recognize their deck of Fraction Cards. If you do not have enough different colors of oak tag for all the decks, each group should mark the backs of all the cards in their deck with an identifying mark or colored scribble to differentiate them from the other decks in the class.

Student Activity Book, p. 27

After students have finished making the fraction cards, on the board we will pose a problem for students to have them start thinking and using the Fraction Cards.

We will pose the problem “A fourth grader was making Fraction Cards for  $\frac{3}{4}$  and  $\frac{5}{8}$ , and said that she noticed something about both of them. She said that they both have ‘one piece missing,’ so they must be the same” (p. 76).

We will ask the students “What do you think she means by ‘one piece missing’ (p. 76)?” [A student might respond with “By ‘one piece missing’ I think she means that there is one part missing from the whole. For example, in  $\frac{5}{6}$  there are five parts shaded on the fraction card, but one is not shaded.”]


We will continue asking students “Do you agree with that? How would you make an argument for whether you agree or disagree with the fourth grader (p. 76)?”

This question will engage the students and get them thinking about comparing two fractions with a “one piece missing.”

***Explore:***

***Making the Fraction Cards***

<b><u>Questions</u> , Problems, or Learning Activities</b>	<b><u>Responses Anticipated from Students</u></b> Include labels: good, partial, or misconceived.	<b><u>Support from Teacher</u></b>	<b><u>Teacher Notes</u></b>

<p>(Write brief description of task 1.)</p> <p>How would you create a fraction card for the fraction <math>5/3</math>?</p>	<p>A correct response with good reasoning.</p> <p>I know that the fraction <math>5/3</math> is divided into equal parts of thirds. But the numerator is greater than the denominator which tells me that I will have more than a whole. I know that a whole is <math>3/3</math> but I still need <math>2/3</math> to represent <math>5/3</math>. So I will need to draw a whole and <math>2/3</math>. In order to draw a whole I will create a fraction card by equally dividing a square into 3 equal parts and shade all the parts. Then, I will draw another square and divide into 3 equal parts, but only shade 2 parts of the 3 parts. So, I have <math>3/3 + 2/3 = 5/3</math> and I can see that using my fraction cards.</p>  <p>(Note: We could not shade the boxes so we used x to show the shaded parts).</p>	<p>Do you think you can rewrite <math>5/3</math> into a mixed fraction? If yes, how? If not, no why? How can rewriting this fraction into a mixed fraction help you?</p>	<p>Student understands how to decompose a fraction into 2 fractions. He decomposed the fraction <math>5/3</math> into <math>3/3</math> and <math>2/3</math> to make it easier for him to represent it.</p>
	<p>A second correct response with different reasoning.</p> <p>I know that I will have 5 shaded parts because the numerator tells me the number of shaded parts, and I know that I will be dividing each of the squares into 3 equal parts, because the denominator tells me how many parts the whole is equally divided into. Since the numerator is greater than the denominator I know that I will have more than a whole, so I know I will need to draw</p>	<p>How do you know how many wholes you will need? How did you know you needed two wholes and not one or three wholes? Decompose the fraction and show me how rewriting the fraction into two fractions can help you identify how many wholes you will need to have.</p>	<p>Student understands the meaning of numerator and denominator, and knows how to use that to make a representation of his fraction using fraction cards.</p>




	two boxes in order to shade 5 thirds.		
	<p><i>Minimal, partial, or incomplete response.</i></p> <p>I will draw 2 boxes and shade 5 parts.</p>	<p>You said you're going to draw 2 boxes. How many parts will each box be divided into? Which number in the fraction tells you how many parts to divide the box into?</p>	<p>Student needs to specify which part of the fraction tells him how many parts of the whole to shade.</p> <p>Student needs to relate the numerator to the number of parts he will divide the whole into. Student was not specific about how many parts to divide the wholes into.</p>
	<p><i>Misconceived or incorrect response (involves a misconception or notable mathematical error)</i></p> <p>I can draw a box and divide it into five equal parts. Then shade three of the parts.</p>	<p>Is the 5 in the numerator or denominator? What does the denominator tell you, and what does the numerator tell you?</p>	<p>The student does not have an understanding of what the numerator and the denominator represent.</p>

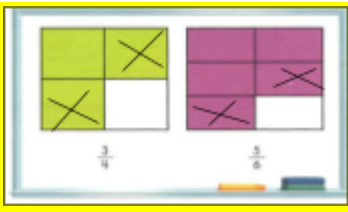
<p>How would you make the Fraction Card for <math>0/4</math>?</p>	<p><b>A correct response with good reasoning.</b></p> <p>I have zero dollars. There are four people. How many dollars can I share with them?</p> <p>This will help me with creating the Fraction card because I know that there are four people, so I need to divide the box into four parts. Since I don't have any money I cannot shade any of the boxes.</p> <p><b>A correct response with good reasoning.</b></p> <p>The denominator tells us how many parts to divide the whole. The denominator is four so I divided the box into four equal parts. The numerator tells us how many parts to shade. Since it's zero we don't shade any parts.</p> <p><b>Minimal, partial, or incomplete response.</b></p> <p>I know that zero divided by four is zero.</p>	<p>What does not shading any of the boxes mean? How can you use your representation to find how much <math>0/4</math> is?</p> <p>I understand from what you are saying, that for any fraction that has zero as a numerator, we do not need to shade any parts of the whole? If you had <math>0/6</math> would you shade any parts of the whole?</p> <p>Why is dividing zero by any number a zero? How would knowing that zero divided by four is zero help you when creating your Fraction card?</p>	<p>Student is able to create a word problem to reason through how to create his fraction card.</p> <p>The student understands that the denominator is the number of parts the whole is divided into, while the numerator is the number of shaded parts.</p> <p>The student understands what the denominator and numerator represents. The student knows how to use the meaning of the numerator and denominator to create a fraction card.</p> <p>The student can relate fraction to division, however he does not understand how to use that to help him determine how many equal parts to divide the whole into, and how many parts to shade.</p>
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	<p><b>Misconceived or incorrect response (involves a misconception or notable mathematical error)</b></p> <p>The numerator is zero, so I am not able to create a box for my fraction card to represent the fraction <math>0/4</math>, because my answer will be 0, and zero means nothing.</p>	<p>Does zero in the numerator indicate that you do not have any shared parts and you do not divide the whole into any parts? If so, what does the 4 tell you?</p>	<p>Student does not understand the meaning of the numerator and denominator.</p> <p>Student thinks about zero as nothing, so she relates to zero as not being able to draw anything.</p>
<p><i>How would you make the Fraction Card for <math>1\frac{1}{3}</math>?</i></p>	<p><b>A correct response with good reasoning.</b></p> <p>I know that 1 in this mixed fraction represents a whole which is <math>3/3</math>, but my fraction is a whole and <math>\frac{1}{3}</math>, so I will need more than one box to be able to represent the whole fraction. I will draw two boxes and divide each into 3 equal parts, because I know I am working with thirds. I will shade a whole box because I have 1 whole and I will shade 1 part of the second box because I have <math>\frac{1}{3}</math>.</p> <p><b>A correct response with good reasoning.</b></p> <p>I know that 1 in this fraction represents <math>3/3</math>, and I also have <math>\frac{1}{3}</math>, so <math>3/3 + \frac{1}{3} = 4/3</math>.</p> <p>Now, I know that I will need two boxes and I will divide each box into three parts. I will shade 3 parts in the first box, and 1 part in the second box.</p>	<p>You said you will divide the boxes into three parts because you are working with thirds. How do you know that you are working with thirds?</p> <p>How do you know that you will have two boxes?</p> <p>Do you divide the box into random parts, or do the 3 parts need to be divided in a certain way? {example of response equal parts}</p>	<p>The student has a clear understanding of a mixed fraction.</p> <p>The student understands what 1 in a mixed fraction represents.</p> <p>The student can make connections between whole numbers and fractions.</p> <p>The student knows how to convert from mixed fraction to improper fraction.</p> <p>The student knows how to add fractions with the same denominator.</p> <p>The student is able to represent a fraction that is</p>

			greater than a whole using fraction cards.
	<p><b>Minimal, partial, or incomplete response.</b></p> <p>I will draw one box and not divide it into any equal parts because I have 1. I will shade it all. Then, I will draw another box and divide into 3 parts and shade one part.</p>	<p>What does 1 in the mixed fraction <math>1\frac{1}{3}</math> represents? [example of response 3/3].</p> <p>How can you use your knowledge about what 1 represents in this fraction to create your fraction card?</p>	<p>Student has an understanding that 1 means shading the whole thing, but does not know how to convert the 1 whole into a fraction to be able to represent it using fraction cards. In this question, 1 converted into a fraction is 3/3.</p>
	<p><b>Misconceived or incorrect response (involves a misconception or notable mathematical error)</b></p> <p>To create the fraction card, I started by adding the fractions. I will add the numerators which are 1+1 and will get 2. The fraction becomes 2/3. I drew a box and divided it into three equal parts because of the denominator. Then I shaded two parts.</p>	<p>What does the 1 in this mixed fraction mean? In order to add fractions we need to have a common denominator right? What is the common denominator that you are using to add <math>1+\frac{1}{3}</math>?</p>	<p>The student does not know what the 1 in the mixed fraction <math>1\frac{1}{3}</math> represents.</p> <p>The student mistakenly adds <math>1+\frac{1}{3}</math> and gets <math>\frac{2}{3}</math>.</p> <p>The student has a misunderstanding when converting a mixed fraction into an improper fraction.</p>

## One Missing Piece

<b>Questions, Problems, or Learning Activities</b>	<b>Responses Anticipated from Students</b> Include labels: good, partial, or misconceived.	<b>Support from Teacher</b>	<b>Teacher Notes</b>
<p>(Write a brief description of task 1.)</p> <p>Students will take out the Fraction Cards that represent <math>\frac{3}{4}</math> and <math>\frac{5}{6}</math>.</p>  <p>With a partner, students will determine and come up with an argument whether <math>\frac{3}{4}</math> and <math>\frac{5}{6}</math> are the same size or whether one is larger than the other. Students will use drawings and further examples to support their argument.</p>	<p>A correct response with good reasoning.</p> <p>I think <math>\frac{5}{6}</math> is not the same size as <math>\frac{3}{4}</math>. <math>\frac{5}{6}</math> is greater than <math>\frac{3}{4}</math> because if we look at the Fraction Cards for <math>\frac{3}{4}</math> and <math>\frac{5}{6}</math>, the missing piece in <math>\frac{3}{4}</math> which is <math>\frac{1}{4}</math> is greater than the missing piece in <math>\frac{5}{6}</math> which is <math>\frac{1}{6}</math>. Since <math>\frac{1}{6}</math> is smaller than <math>\frac{1}{4}</math>, <math>\frac{5}{6}</math> is closer to one whole, so it is the greater fraction.</p>	<p>How do you know that <math>\frac{1}{4}</math> is greater than <math>\frac{1}{6}</math>? How does that help you figure out which fraction is greater?</p>	<p>The student has a good understanding of using the missing piece to compare two fractions.</p> <p>The student was able to use the distance from a whole to compare <math>\frac{3}{4}</math> and <math>\frac{5}{6}</math> by looking at the missing piece in each fraction and their distance away from the whole.</p> <p>Next: plan more challenging questions for the student to work with. Have the student compare 3 fractions instead of 2.</p>
	<p>A second correct response with different reasoning.</p> <p>I think <math>\frac{3}{4}</math> and <math>\frac{5}{6}</math> are not the same size. <math>\frac{5}{6}</math> is greater than <math>\frac{3}{4}</math> because if I take two more parts from the shaded part of <math>\frac{3}{4}</math> I will be left with <math>\frac{1}{4}</math>, but if I take two of the shaded parts from <math>\frac{5}{6}</math> I will be left with <math>\frac{3}{6}</math> or <math>\frac{1}{2}</math>.</p>	<p>Why is it that even after taking the same number of shaded parts from both fractions, one is larger than the other?</p> <p>Hint: Think about the size of the pieces. What happens to the size of a whole when you divide it into more parts?</p>	<p>The student is able to create easier fractions to make the comparison easier, by subtracting the same amount of parts from each fraction. For example, the student took two parts away from <math>\frac{3}{4}</math> and <math>\frac{5}{6}</math> to make <math>\frac{1}{4}</math></p>

	<p><math>\frac{3}{4} - \frac{2}{4} = \frac{1}{4}</math>  <math>\frac{5}{6} - \frac{2}{6} = \frac{3}{6}</math> or <math>\frac{1}{2}</math></p>  <p><math>\frac{5}{6}</math> is greater than <math>\frac{3}{4}</math> because there are more pieces left after taking two pieces away from the shaded parts of both fractions.</p>		<p>and <math>\frac{3}{6}</math> and used that to compare which fraction is greater.</p> <p>The student is able to rewrite the fraction <math>\frac{3}{6}</math> to its equivalent <math>\frac{1}{2}</math> to make the comparison with <math>\frac{1}{4}</math> easier.</p>
	<p><i>Minimal, partial, or incomplete response.</i></p> <p>If we look at the Fraction Cards for <math>\frac{5}{6}</math> and <math>\frac{3}{4}</math> more is shaded in <math>\frac{5}{6}</math> than in <math>\frac{3}{4}</math>, so <math>\frac{5}{6}</math> is bigger.</p>	<p>How about if you don't have the drawing or Fraction cards, how can you determine if <math>\frac{5}{6}</math> is less than, equal to or more than <math>\frac{3}{4}</math>? How can the size of the "one missing piece" help you compare the fractions?</p> <p>Hint: Think about the distance each fraction is away from the whole.</p>	<p>The student is only relying on the drawings to compare the 2 fractions without much reasoning.</p> <p>Work with the student one on one to help him/her be able to incorporate their knowledge of the meaning of fractions and their parts to be able to compare different fractions.</p>
	<p><i>Misconceived or incorrect response</i> (involves a misconception or notable mathematical error)</p> <p>The numerator and denominator in <math>\frac{5}{6}</math> are bigger than the numerator and denominator in <math>\frac{3}{4}</math>, so <math>\frac{5}{6}</math> is greater than <math>\frac{3}{4}</math>.</p>	<p>In order for one fraction to be greater than the other does the numerator and denominator have to be greater?</p> <p>(Hint: think about the meaning of numerator and denominator when comparing fractions).</p>	<p>Provide the student with different fractions that will expose his/her misconception.</p> <p>For example, by looking at the fractions <math>\frac{1}{2}</math> and <math>\frac{2}{6}</math> the student will be able to recognize that the greater numerator and</p>

			denominator do not always indicate the greater fraction. In $\frac{2}{6}$ the numerator and denominator are greater than the numerator and denominator in $\frac{1}{2}$ but $\frac{1}{2}$ is greater than $\frac{2}{6}$ .
Unanticipated Responses observed during the lesson.	[For when you really teach the lesson, not for this assignment!]		

**Formative Assessment:** (What will you look or listen for to know students are making good progress?)

While students are discussing their arguments we will walk around and listen to students' responses. While walking around the classroom we will keep in mind the following questions and take notes based on our observations:

- "How do students compare fractions that are one fractional part less than 1? Do they begin with the understanding that both wholes in this case must be the same size? Do they try to keep the sizes of the parts the same and increase the size of the whole (p. 77)?"
- "Do they understand that  $\frac{1}{6}$  and  $\frac{1}{4}$  are different parts of the whole even though each is "one piece" of the whole? Do they see that the more equal parts you cut the whole into, the smaller each part? Do they think that sixths are bigger than fourths because the value of 6 is greater than that of 4 (p. 77)?"
- "Can students make arguments based on their knowledge of the meaning of fractions, or do they rely on their drawings, which may or may not be accurate" (p. 77)?"

We will listen to students' responses during the whole class discussion. In addition, we will also look at the results from the exit ticket that we will give at the end of the lesson.

### **Share and Summarize:**

#### **a. What 2 or 3 solutions will be shared and in what order?**

Why did you choose those solutions and that sequence?

The two solutions the whole class will discuss together are how to represent  $1\frac{1}{3}$  using fraction cards and how to compare  $\frac{1}{4}$  and  $\frac{1}{6}$ . We will begin by discussing how to represent  $1\frac{1}{3}$  using Fraction cards because according to the article "Equal Sharing Problems and Children's Strategies for Solving Them" by Empson and Levi students tend to think that fractions are between 0 and 1, such as  $\frac{1}{4}$  and  $\frac{1}{2}$ . By starting with a mixed fraction not only will students be exposed to mixed fractions, but they will also be able to recognize that fractions do not necessarily have to be between 0 and 1. This will allow us teachers to evaluate students' ability to make connections between whole numbers and fractions. In addition, we will be able to see if

students are able to convert mixed fractions into improper fractions. The mixed fraction will enable us to determine if students are able to recognize that the whole in  $1\frac{1}{3}$  is  $\frac{3}{3}$ , since we are working with thirds. The second solution we will be discussing as a whole group is comparing  $\frac{1}{4}$  and  $\frac{1}{6}$ . Students compared  $\frac{1}{4}$  and  $\frac{1}{6}$  as the missing parts of a whole when comparing  $\frac{3}{4}$  and  $\frac{5}{6}$ . Some students had a hard time making the connection that since  $\frac{1}{4}$  is greater than  $\frac{1}{6}$ ,  $\frac{5}{6}$  is closer to the whole because the distance from  $\frac{5}{6}$  to  $\frac{6}{6}$  is less than the distance from  $\frac{3}{4}$  to  $\frac{4}{4}$ . Allowing students to compare the fractions as a whole group will allow for the clarification of any misconception and allow us the teachers to relate this comparison to the missing part of the wholes in the fractions we compared earlier in this lesson ( $\frac{3}{4}$  and  $\frac{5}{6}$ ).

***b. What will you focus on in the last part of the lesson? How will you check that students are getting the key ideas? Use the following grid.***

Summary Questions (2-4 Summary Qs)	Responses Anticipated from Students	Support from Teacher
<p>One of your peers stated the following when creating a Fraction Card for <math>1\frac{1}{3}</math>. Is she correct?</p> <p>To create the fraction card, I started by adding the fractions. I will add the numerators which is <math>1+1</math> and will get 2. The fraction becomes <math>\frac{2}{3}</math>. I drew a box and divided it into three parts because of the denominator. Then I shaded two parts.</p>	<p>Good response</p> <p>No, she's not correct because we have a one whole or <math>\frac{3}{3} + \frac{1}{3}</math> is <math>\frac{4}{3}</math> and not <math>\frac{2}{3}</math>. Then, when creating the fraction card she will need to create 2 boxes and divide each into 3 equal parts, shade one whole box to represent the whole and shade 1 part of the second box to represent <math>\frac{1}{3}</math>.</p>	<p>Can you explain to your classmates how you know that a whole in this fraction is <math>\frac{3}{3}</math>? How do you know that you can not add <math>1+1</math> and have the answer <math>\frac{2}{3}</math>?</p>
	<p>Partial response</p> <p>I think that she is incorrect because the whole in <math>1\frac{1}{3}</math> is <math>\frac{3}{3}</math>. so, we must add <math>\frac{3}{3}</math> and <math>\frac{1}{3}</math>. If we add the two fractions we get <math>\frac{4}{3}</math>. The next step is to draw a box and divide it into four equal parts. Then shade three parts of it.</p>	<p>Support for partial response</p> <p>Does the numerator 4 in <math>\frac{4}{3}</math> tell us how many equal parts to divide the box into or how many equal parts to shade? What does the 3 in the denominator represent? What does having a greater number in the numerator than in the denominator tell us about the</p>



		number of boxes we need to draw?
	<p>Misconceived response</p> <p>I think that the student forgot to add the denominators. A whole is <math>1/1</math>, and <math>1/1 + 2/3 = 3/4</math>. When creating the fraction card the student had to divide the box into 4 equal parts and shade 3 of those parts.</p>	<p>Support for misconceived response</p> <p>Can we add fractions that do not have the same common denominator?</p> <p>When adding fractions do we add the different denominators?</p> <p>Does 1 in this fraction represent <math>1/1</math>?</p>
Remember earlier in this lesson when comparing $3/4$ and $5/6$ we looked at the one missing part away from the whole. Some of you compared $1/4$ and $1/6$ . Explain how comparing those fractions help you find which fraction is greater: $3/4$ or $5/6$ .	<p>Good response</p> <p><math>1/6</math> is smaller than one fourth because if I have one piece of brownie, I would rather share it with 4 people than with 6 people.</p>	<p>Why would you rather share with 4 people and not 6? What happens to the size of the pieces when you share with more people?</p>
	<p>Partial response</p> <p><math>1/4</math> and <math>1/6</math> have the same numerator, but different denominators. I know that for both fractions I will have only one shaded part. The 6 is 2 parts more than 4 so <math>1/6</math> is greater than <math>1/4</math>.</p>	<p>You said the 6 is 2 parts more than the 4, but are the parts the same size?</p>
	<p>Misconceived</p> <p>The six in the denominator in the fraction <math>1/6</math> is greater than the 4 in the fraction <math>1/4</math>, so <math>1/6</math> must be greater.</p>	<p>Does the greater number in the denominator indicates that the fraction is greater? The denominator indicates how many pieces the whole is divided into. So, if I divide a pizza with 6 friends will they get a larger slice than if I divided it with 4 friends?</p>

**Exit Ticket:** [Provide a relevant, short Q you would ask students to answer in writing before leaving class. Include a good response in brackets.]

Now that students have worked with a partner to come up with an argument why  $\frac{3}{4}$  and  $\frac{5}{6}$  are the same size or whether one is smaller than the other, students will complete the exit ticket individually. On a sheet of paper, students will write their ideas about whether  $\frac{5}{6}$  is less than, equal to or more than  $\frac{3}{4}$ . Students will write their reasoning. They will also include questions they have if they don't understand how to compare the two fractions. We will collect the papers to see how students were reasoning when comparing the two fractions.

[Response:  $\frac{5}{6}$  and  $\frac{3}{4}$  are both one missing part away from the whole.  $\frac{5}{6}$  is  $\frac{1}{6}$  away from the whole while  $\frac{3}{4}$  is  $\frac{1}{4}$  away from the whole. I know that  $\frac{1}{4}$  is greater than  $\frac{1}{6}$  because 1 is partitioned into 4 equal parts in  $\frac{1}{4}$  and into 6 equal parts in  $\frac{1}{6}$ . The 4 equal parts are larger than the 6 equal parts which tells me that  $\frac{1}{4}$  is greater than  $\frac{1}{6}$ . So, since the missing part  $\frac{1}{4}$  is greater than the missing part  $\frac{1}{6}$  this tells me that the distance  $\frac{3}{4}$  is away from the whole is greater than the distance  $\frac{5}{6}$  is away from the whole. Thus,  $\frac{5}{6}$  is greater than  $\frac{3}{4}$  because it is closer to 1.]

Question: I would like to know if this reasoning works to compare fractions more than one missing part away from the whole. If we are comparing fractions that are two or more missing parts away from the whole can we still think about how close the missing parts are away from the whole to find out which fraction is greater?]

Note: The lesson on TERC session 2.2, asks students to compare  $\frac{3}{4}$  and  $\frac{5}{6}$  individually as an exit ticket.

### Other

#### References

- TERC Investigations Grade 4 U6 Ordering Fractions 2.1
- TERC investigation Grade 4 U6 Ordering Fractions 2.2
- Houghton Mifflin Company (2007). Houghton Mifflin Math. Grade 4. Boston, MA: Houghton Mifflin Company.
- Common Core State Standards: Mathematics Standards. (2010). December 1, 2021. Retrieved from <http://www.corestandards.org/Math/>.
- *Principles and Standards for School Mathematics*. National Council of Teachers Mathematics Inc. (2000), Number and Operations. Standards for Grades 3–5. 148-156. United States of America. Copyright 2000. December 1, 2021.
- Empson and Levi *Equal Sharing Problems and Children's Strategies for Solving Them*. (pgs. 225 - 237)

#### Reflection by each partner

**Be sure you have attached any handouts, transparencies, other materials or photos or descriptions of them.**