Cycling Research Study Pre Plan

Background

The power-duration curve of athletic performance is determined by the maximal work that can be performed over a given length of time. The modern models of the power duration relationship stem predominantly from the Critical Power (CP) Model (Monod and Scherrer 1965, Moritani 1981) . This model states that the curve is determined by the sum of two contributing components; CP which is a power that can be sustained without fatigue for a very long period of time, and a W' which is a finite work capacity that is instantaneously available:

$$P(t) = W'/t + CP$$

This mathematical model produces a hyperbolic curve which has been successfully fit in a broad range of applications. The model however fails outside a relatively narrow time range. Very short durations produce predictions of infinitely high rates, and long durations produce predictions of infinitely sustainable rates (Morton, 2006)..

Attempts to extend the model have included short duration rate limits imposed by linear and exponential feedback mechanisms.

$P(t) = W' * [t - (W'/(CP - Pmax))] + CP$ (Morton, 1996)

$P(t) = \int_0^t [Pmax * e^{(-t/tau)} + CP * (1 - e^{(-t/tau)})]dt$ (Ward-Smith, 1985)

These modifications improve the predictive power at short duration but do not address the CP model failure and long durations.

Logarithmic and exponential rate decays have thus been applied to the CP components in attempts to further extend the model for long durations.

$P(T) = [S/T * (1 - e^{(-T/20)})] + 1/T \int_0^T [BMR + B * (1 - e^{(-t/30)})]dt,\ (T < 420s)$

or

$P(T) = [A + [Afln(T/420)]/T * (1 - e^{(-T/20)})] + 1/T \int_0^T [BMR + [B + E * ln(T/420)] * (1 - e^{(-t/30)})]dt$

$,\ (T > 420s)$ (Peronnet and Thibault, 1989)

$$P(t) = Pmax * [\,tau1\,/\,t * (1 - e^{(-t/tau1)}) - tau1a\,/\,t * (1 - e^{(-t/tau1a)}) +$$
$$CP * [\,tau2\,/\,t * (1 - e^{(-t/tau2)}) - tau2a\,/\,t * (1 - e^{(-t/tau2a)})]$$

(Alvarez, 2002)

These extended models allow better fits across the entire curve but at the expense of significant complexity and reliance on a priori fixed parameters for solvability which may not generalize well for some individuals.

In practice, the original critical power model is still often favored over the extended range models. It appears that extended range often does not outweigh the appeal of a simple intuitive model free of a priori fixed parameters.

To address these pragmatic barriers a new model is proposed here that employs the intuitive CP concept of finite capacities with a simple linear feedback. This novel two component model offers an extended range, minimized complexity, and intuitive parameters.

The new model is derived as follows:

First, let P(t) represent the average power at any duration t as the sum of two contributing components. Let W' represent a finite capacity in joules where W'1 represents a low capacity anaerobic component and W'2 represent a high capacity aerobic component.

$$P(t) = W'1/t + W'2/t$$

Compared to the CP model W' is conserved as W'1. However, CP as an infinitely sustainable rate is replaced by a second component with a limited capacity. The repetitive structure of the model is in keeping with the goals of simplicity and intuitiveness as the capacity limited structure of the first component is carried over to the second component as well.

Next, a simple linear feedback is applied to both components to reflect a rate limitation for each system. Let tau represent a time constant that describes the rate limitation of each component as a function of time. The final equation now becomes:

$$P(t) = W'1/(t + tau1) + W'2/(t + tau2)$$

This equation is the simplest mathematical description of the extended power-duration curve currently proposed.

The equation can also be rewritten in terms of maximal rates. Let Pow represent the maximal rate of each component so that the capacity of the component equals the maximal rate times the time constant:

$$W' = Pow * tau$$

Substitution yields:

$$P(t) = Pow1 * tau1/(t + tau1) + Pow2 * tau2/(1 + tau2)$$

The power curve is now fully described by two components, each with a limited capacity and maximal rate related by a time constant tau. The model thus describes the complete curve as a continuum of sequential regions that can be quantified by the anaerobic rate, anaerobic capacity, aerobic rate, and aerobic capacity parameters respectively.

Problem Statement:

1. Current mathematical models of the power duration relationship are limited in applied use due to one or more of the following factors:
   - narrow ranges of validity
   - excessive model complexity
   - modelling biases
   - incomplete quantification of the power duration curve
   - reliance on empirically derived fixed parameters

Aims:

1. Propose a simple mathematical model that is valid across the full spectrum of the power duration curve.
2. Validate this model by the evaluation of model fit and external validation against laboratory measured performances.

Works Cited

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