Gravity Homework

(Solution steps below)

1. Two 25 kg point masses are placed in a coordinate plane that is marked off in meters, one at (0, 4), and other at (3, 1). What should be the coordinates of a 40 kg mass so that the gravitational field at (3, 4) is zero? (No other masses are in the vicinity.) *Answer: (5.256, 6.256)*

2. Clearly describe the Cavendish experiment in your own words: what it accomplished and how it worked. Explain why Cavendish needed or didn't need to know the length of the rod connecting the two small masses.

3. Planet Gorgonzola orbits at a distance x from its star in a nearly circular orbit, completing a revolution in a time y. Calculate the mass of the planet and its star, or explain why it is not possible to do so.

4. Describe the orbit of a typical comet and explain fully how & why its speed changes.

5. Planet Calamari has a mass of 6.21×10^{25} kg and a radius of 8.9×10^{6} m. Calculate:
   a. the acceleration due to gravity on its surface. *Answer: $52.29\ m/s^2$*
   b. the range of a mango fired horizontally from a 10 m high hill at 500 m/s. *Answer: 309.23 m*
   c. the launch speed necessary so that the mango never lands (that is, goes into orbit).
   Answer: 21,572.7 m/s ≈ 22,000 m/s

6. There are two reasons why Saturn's period is greater than Jupiter's. What are they?

7. Draw the gravitational field lines (which can be curves) for the two-mass system below. *Hints: 1. The net force of gravity at any point is always tangent to the field line at that point. 2. Field lines never cross. (Intersecting field lines would mean the net force on the mass is in two different directions at once.) 3. Fields have direction! Your field lines should have arrowheads on them.*

Solution steps:

1. It may be helpful to imagine that we're in outer space, and there is essentially no gravitational field from anything except for the two small bodies stated in the problem--perhaps two lonely, dinky asteroids. In order to have no gravitational field at the point P = (3, 4), we need to figure out how strong and which way the gravitational field is due to the given masses, then place a 40 kg mass in an appropriate location, Q, so as to cancel out the field the other two masses produce. Let's imagine an arbitrary mass m placed at P. P happens to be 3 meters from each of the other masses, and since they're each 25 kg, they each exert a force on m of F = G m (25) / 3^2 = (25/9) G m. These two forces are perpendicular and their vector sum (let's call it **F1)** is down and to the left at a 45 deg

angle. If you make a picture of this, you'll see you have a 45-45-90 triangle. Each side is F, and the hypotenuse is F1 = F sqrt(2) = (25/9) G m sqrt(2). The 40 kg mass must pull just as hard on m but in the other direction, from an unknown distance r. Let's call this force F2, and we have F2 = G m (40) / r^2. In order to cancel each other out, **F1** = -**F2**, and F1 = F2. So, equate them and solve for r. You should get a little over 3 m. Thus Q must be a distance r away from P, up and to the right at 45 deg. The x-coordinate of Q is the x-coord of P + r cos(45 deg). Find the y-coord of Q in a similar way.

2. Check out the slides on the Cavendish experiment. Remember that even a small force can produce a large torque if its moment arm is long enough. However, because the universal gravitational constant, G, is so small, the force of gravity between two ordinary-sized objects is generally itsy bitsy. So, any angular displacement of a hanging rod due to gravitational forces is miniscule, even with a very long rod. The longer the rod, the more likely that displacement will be measurable.

3. According to Kepler's third law, the square of the period of a planet is proportional to the cube of its mean distance from the sun. That is, T^2 =kR^3, where k is a constant of proportionality. For circular orbits, this is proven by noting that it is the gravitational force on the planet that provides the requisite centripetal force to circle the sun. So, GMm / R^2 = mv^2 / R, from which we see that m, the planet's mass, cancels. This shows that the orbital speed and period of a planet in a circular orbit are independent of its mass. (A speck of dust 93 million miles from the sun will circle it in one year, just like earth does.) However the mass of the star does matter. Simplify the equation above by cancelling m and 1/R. Since the orbit is circular, the speed is constant, so v = 2 pi R / T, where T is the period. Make this substitution and solve for T^2. You should get T^2 = (constant) R^3. The constant should contain G and M. You're given the values for T and R in terms of x and y. Solve for M in terms of x and y.

4. Many comets have very eccentric orbits. This means their orbits, though still elliptical, are far from circular. The foci of the ellipse are far apart, almost as far apart as the vertices. The sun, as always, is one focus. So, sometimes a comet is very close to the sun, and sometimes it's very far. Draw such an ellipse, mark the sun, and put the comet on the ellipse far from the sun but heading toward it. Draw the force of gravity (point straight toward the sun). Since the orbit is not circular, the force vector is not purely radial. It's also partly tangential. Break the force into components, one tangent to the ellipse, and one perpendicular to it. The radial component is the centripetal force, which causes the comet to have a curved path. The tangential component should be pointing in the direction of motion, and it causes the comet to speed up as it approaches the sun. Now draw the planet just as far from the sun but heading away from it. Draw the force vector and its comps. This time you should see that the tangential comp is in the opposite direction of motion, slowing the comet down. Finally, to illustrate Kepler's second law, from the sun draw two "elliptical sectors" of equal area, one on each side of the sun. One should be short and stout, the other long and skinny. Kep's 2nd law asserts equal areas in equal times. On the short, stout sector mark endpoints A and B. Call the endpoints of the long, skinny sector C and D. Since the two sectors have the same area, the planet should travel from A to B in the same time as from C to D. Notice, though, that C and D are pretty close together, meaning the planet moves slowly way out there. This is consistent with our earlier analysis involving forces and components.

5. a. Weight is, of course, mg. Unlike big G, little g is not a universal constant; it's only constant for a given planet (and even then it can fluctuate a bit). Weight is the force of gravity, so it can also be

expressed with Newton's law of gravitation. Equating the two forms yields $mg = GMm / R^2$, where M is the mass of the planet, R is its radius, and m is the mass of some arbitrary object on the planet's surface. Since m can be canceled, we see that the acceleration due to gravity (a.k.a. the gravitational field strength) does not depend on the mass placed in the gravitational field. Rather, it depends only on location and the body producing it (the planet). Anyway, you can solve this equation for g and plug in the given values for M and R. By the way, if you do this with earth's mass and radius, you get 9.8 m/s^2, which is consistent with experiment.
b. Now that you know g for this planet, we just do projectile motion. 500 m/s is pretty fast, but it shouldn't be fast enough to account for the fact that the planet is spherical. Recall that when we do projectile motion, we are essentially assuming a flat earth, i.e., a uniform gravitational field (constant strength with parallel field lines). First find the hang time using the delta x equation with $v0 = 0$ (since there's no initial vertical vel) and $a = -g$ (using the value from part (a)). Horizontally there is no acceleration, so it's just $d = vt$.
c. In order to circle the planet, a projectile needs a centripetal force. This comes from gravity, and its magnitude is known: g. Centripetal accel = v^2 / R. So, set this equal to g and solve for v.

6. Which planet lies farther from the sun and, hence, must travel a greater distance around it? Also, the farther from the sun, the weaker its gravitational field. Remember, gravitational field is measured in N/kg (it's force per unit mass). But 1 N/kg = 1 (kg m/s^2) / kg = 1 m/s^2. So gravitational field strength is really just acceleration due to gravity. Thus planets further from the sun have less centripetal acceleration. Look at the formula for centripetal accel. If it is smaller, what does that mean for speed?

7. Let's begin by looking at the perpendicular bisector of the segment connecting the two masses. Pick any point P on it. Since the masses are equal and P is equidistant to each mass, the forces are the same. Draw the force vectors. The vector sum (the net force) should lie on the perp. bisector. This would be true for any point on the bisector. Recall that at any point the net force is always tangent to the field there. This means that one field line is the bisector you've drawn (except for the point halfway between the masses, since the force there is zero. Use similar reasoning to find that the line connecting the masses is also a field line. The rest are curves. Here's why. First, think about points very close to one of the masses. Here the other mass's gravitational influence is negligible, so the field near this mass is as if the mass were isolated. An isolated point mass has a field with radial symmetry. Draw fields like this near each mass. Farther out, both masses matter. If the radial field lines from one mass continued indefinitely, they'd cross those from the other (along with the line that lies on the bisector). This cannot be the case, since intersecting field lines would imply two different directions for the net force. Instead, what happens is that the field lines begin as radial lines near the masses and then veer off to the outside, as if they're repelling each other. Those heading toward the bisector zoom up (or down) toward infinity. The bisector is like a vertical asymptote. Lines coming off the left mass veer off to the left, likewise for the right side. To understand this better, pick a Q somewhere directly above the left mass. The force on it due to the left mass is straight down, and the force on it due to the right mass is toward it--down and to the right. The latter force is weaker, due to greater distance, so this vector is shorter. Use the parallelogram method to add the vectors. This Fnet vector points somewhere between the other two and shows the direction of the field at Q. It should be oriented downward and slightly to the right. Make sure your field line is tangent to **Fnet** at Q. Do you see why the field curves outwards now? Make sure your field lines have arrows on them (pointing toward the masses).