Unit 08: Differentiation Rules

Unit Objectives

- Learn the shortcuts for determining derivatives
  - Purchase coffin
  - Insert $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
  - Bury deep in an abandoned field
- Find higher order derivatives
- Find derivatives of inverse functions

Unit 08 Lesson 01: Differentiate Polynomials

- Determine derivatives of polynomials using the Power Rule

~~~~~~

Can you spot the pattern?

~~~~~~

1. $\frac{d}{dx} 5x^3 =$
2. $\frac{d}{dx} 3x =$
3. $\frac{d}{dx} 3 =$
4. $\frac{d}{dx} \frac{2}{x} =$
5. $\frac{d}{dx} (2x^2 - \frac{5}{x^3}) =$
6. $\frac{d}{dx} (6\sqrt{x}) =$

~~~U8L1 Homework~~~ 3.3: 1-6

~~~U8L1 Classwork~~~

7. $\frac{d}{dx}(\frac{1}{6}x^2 - x) =$

8. $\frac{d}{dx}3x^4 =$

9. $\frac{d}{dx}x =$

10. $\frac{d}{dx}0 =$

11. $\frac{d}{dx}\frac{5}{x^2} =$

12. $\frac{d}{dx}(-\frac{5}{x} + 2x^2) =$

13. $\frac{d}{dx}\frac{1}{6\sqrt{x}} =$

14. The height of a ball dropped from a 1000ft tall building can be found using the $y = -16t^2 + 1000$.
    a. Find the instantaneous velocity of the ball when it has been falling for 3 second.

    b. Find the instantaneous velocity of the ball when it has been falling for 4 seconds.
~~~

Unit 08 Lesson 02: Identify Compositions

Lesson Objectives

- Find a way to express a function as a composition of two functions

Identify f(x) and g(x) such that

- $h(x) = f(g(x))$ and
- f(x) and g(x) are functions you know how to differentiate.

Then determine f'(x) and g'(x) just for practice.

1. $h(x) = (4 - 3x)^3$

2. $h(x) = (x^2 - 2x)^2$

3. $h(x) = \sqrt{2x - 1}$

4. $h(x) = \left|\frac{x}{3}\right|$

5. $h(x) = \dfrac{2}{x - 1}$

6. $h(x) = (x+2)^3 - (x+2)^2 + 2(x+2) - 5$

~~~Homework U8L2~~~ Nothing
~~~Classwork U8L2~~~

Identify f(x) and g(x) such that

- h(x) = f(g(x)) and
- f(x) and g(x) are functions you know how to differentiate.

Then determine f'(x) and g'(x) just for practice.

1. h(x) = $\sqrt{2x - 9}$

2. $h(x) = \frac{1}{x^2 - x}$

3. h(x) = $(-4 + \sin(x))^2$

4. $h(x) = ln(\frac{3}{x})$

5. $(x-3)^2 - (x-3) + \sqrt{x - 3}$

6. $h(x) = |x^4 - x|$

Unit 08 Lesson 03: Differentiate Compositions (Chain Rule)

Lesson Objectives

- Use the chain rule to differentiate composite functions

Chain Rule:

Determine $\frac{dy}{dx}$.

1. $y = (4 - 3x)^3$

2. $y = \sqrt{4 - x^3}$

3. $y = \frac{4}{3x^2 + 1}$

4. $y = (x+2)^3 - (x+2)^2 + 2(x+2) - 5$

5. $h(x) = \frac{1}{x^2 - x}$

6. $h(x) = \sqrt{2x - 9}$

~~~Homework U8L3~~~ 4.1: 13, 19

~~~Classwork U8L3~~~

Determine y’

1. $y = \sqrt{2x - 1}$

2. $y = (x^2 - 2x)^2$

3. $y = \dfrac{4}{x^2 - x}$

4. $(5x-3)^2 - (4x-3) + \sqrt{2x - 3}$

5. $y = (5(2x^2-1)^2)^4$

6. $y = (4x^3 - 5(2x^2-1))^4$

Unit 08 Lesson 04: Differentiate Exponential Functions

Lesson Objectives

- Apply the differentiation rule for exponential functions

Differentiation Rule for exponential functions

Differentiate the following functions

1. $h(x) = 3(2^x) + 1$
2. $f(x) = 3e^x + 2x + 10$
3. $g(x) = e^{3x} + e^{x/4}$
4. $k(x) = e^{x^2 - x} - 2x + 5$
5. $m(x) = (x + e^{3x})^2$

~~~Homework U8L4~~~ 4.4: 1-5

~~~Classwork U8L4~~~

Differentiate the following functions

1. $f(x) = 4x - 3e^x$

2. $g(x) = 10^x + 5x + 4$

3. $h(x) = 3x^4 - x^2 + e^{-2x}$

4. $r(x) = e^{e^x}$

5. $q(x) = \sqrt{e^{-3x} + 2}$

Unit 8 Lesson 05: Differentiate Logarithmic Functions

Lesson Objectives

- Apply the differentiation rule for exponential functions

Differentiation Rule for exponential functions

Differentiate the following functions

1. $h(x) = 2\log_{10}x$
2. $f(x) = \ln(x+3) - 3x + 10$
3. $g(x) = \ln(4x^2)+\ln(2)$
4. $k(x) = \ln(3e^x) - 2x$
5. $m(x) = (\ln(3x))^2$

~~~Homework U8L5~~~ 4.4: 15-19

~~~Classwork U8L5~~~

Differentiate the following functions

1. $f(x) = x^2 - x + \sqrt{x} - \ln(x) - e^x + 9$

2. $g(x) = -2ln(\frac{1}{x^2})$

3. $h(x) = \sqrt{ln(x)} - 2x + 4$

4. $k(x) = 2ln(2x + 4)$

5. Hmm, if $f'(x) = \frac{1}{2x + 5}$, what's the equation for f(x)?

Unit 8 Lesson 06: Differentiate Trigonometric Functions

Lesson Objectives

- Apply the differentiation rule for sine and cosine functions

Differentiation Rule for sine, cosine, and tangent functions

Differentiate the following equations

1. $y = 2\sin(x) + \tan(x) - 4$
2. $f(x) = 3\cos(2x) + 2x$
3. $g(x) = -\sin(x^2+1) + \ln(x) + e^x$
4. $z = \sin^2 x$
5. $y = \cos^3 x^2$

~~~Homework U8L6~~~ 3.5: 1, 2, 3

~~~Classwork U8L6~~~

Differentiate the following functions

1. $f(x) = \tan(x) - 2x^3 + 1$

2. $y = \cos^2 x$

3. $g(x) = -\sin(2x) - \cos(3x) - x$

4. $h(x) = \sin(\ln(x)) + \ln(x) + 10$

5. $e^{\sin(x)} + 3x^2 - 5$

Unit 8 Lesson 07: Differentiate Products of Functions

Lesson Objectives

- Apply the differentiation rule for two functions multiplied together

Product Rule

Differentiate the following equations

1. $y = x^2 \sin x$
2. $\ln(x)\cos(x) + 2x - e$
3. $g(x) = e^x(x^2 - 2x)^3 - \pi x$
4. $z = \sin x \cos 2x$
5. $y = e^{2x}e^{-3x}$

~~~Homework U8L7~~~ 3.3: 13, 15, 16, 20 just differentiate, ignore stuff about graphically

~~~Classwork U8L7~~~

Differentiate the following functions

1. $f(x) = 2x\cos x + 49$

2. $y = e^x \sin x + ex$

3. $g(x) = \cos^2 x$, solve two ways, a) rewrite as $(\cos x)^2$ b) rewrite as $\cos x \cos x$

4. $e^{-2x}\sin(-3x) - \pi x^2$

5. $e^{x\sin(x)} + 7$

Unit 8 Lesson 08: Differentiate Quotients of Functions

Lesson Objectives

- Apply the differentiation rule for two functions multiplied together

Quotient Rule

Differentiate the following equations

1. $y = \frac{x^2}{sinx}$

2. $y = \frac{cosx}{lnx} + 4x - 2$

3. $f(x) = \dfrac{e^x}{(x^2 - 2x)^2} + e$

4. $f(x) = \dfrac{cos2x}{sinx} + 54321$

5. $g(x) = \dfrac{x^3}{x^2}$, solve by simplifying first, then solve again without simplifying first

~~~Homework U8L8~~~ 3.5: 8,10

~~~Classwork U8L8~~~

Differentiate the following functions

1. $h(x) = \frac{cosx}{2x}$

2. $h(x) = \frac{sinx}{e^x} + e^3$

3. $h(x) = \frac{sinx}{cosx}$ solve by simplifying first, then solve again without simplifying first

4. $y = \frac{e^{2x}}{sinx}$

5. $y = e^{\frac{sinx}{x}}$

Unit 8 Lesson 09: Determine Higher Order Derivatives

Lesson Objectives

- Find second derivatives

Determine y''. Simplify

1. $y = x^4$
2. $y = e^x$
3. $y = \sin x$
4. $y = 3\sin(5x+5)$
5. $y = \dfrac{7}{4x^3}$
6. $y = \sin x \cos x$

~~~Homework U8L9~~~ 3.5: 8,10

~~~Classwork U8L9~~~

Determine y''. Simplify

1. $y = 4x^5$
2. y = cosx
3. y = 4cos(3x)

4. (lnx)/x

6. y = $\sqrt{x}$

Unit 3 Lesson 10: Review for Differentiation Rules Quiz

Lesson Objectives

- Prepare to get a grade on the quiz that would make Mama proud.

1. Differentiate the following functions
    a. $f(x) = 4\sqrt{x}$

    b. $f(x) = xsinx$

    c. $f(x) = cos^3 x$

    d. $f(x) = \frac{lnx}{e^x}$

e. $f(x) = 3\sqrt{x^2 + 4}$

2. For the equation $y = xe^x$ determine y''. Simplify.

3. For the equation $y = \frac{ln x}{x}$ determine y''. Simplify.

Unit 8 Lesson 11: Determine Derivatives at Specific x-Values

1. If $h(x) = x^3$, determine $h'(4)$

2. If $h(x) = x^2e^{2x}\sin(e^x)$, determine $h'(4)$

3. If $h(x) = \sqrt{x^3 - 4}$, determine $h'(3)$

4. If $h(x) = \sin(\cos(\sin(\cos x)))$, determine $h'(1)$

5. If $h(x) = \dfrac{e^{4x}}{x}$, determine $h'(2)$

~~~Homework U8L11~~~ Does anyone read this anymore?

~~~Classwork U8L11~~~

1. If $h(x) = x^4$, determine $h'(2)$

2. If $h(x) = x^2e^{\sin x}\ln(2^x)$, determine $h'(4)$

3. If $h(x) = \sqrt{cos x}$, determine $h'(3)$

4. If $h(x) = ln(\sqrt{sin x^2})$, determine $h'(1)$

5. If $h(x) = \frac{x}{e^{3x}}$, determine $h'(2)$

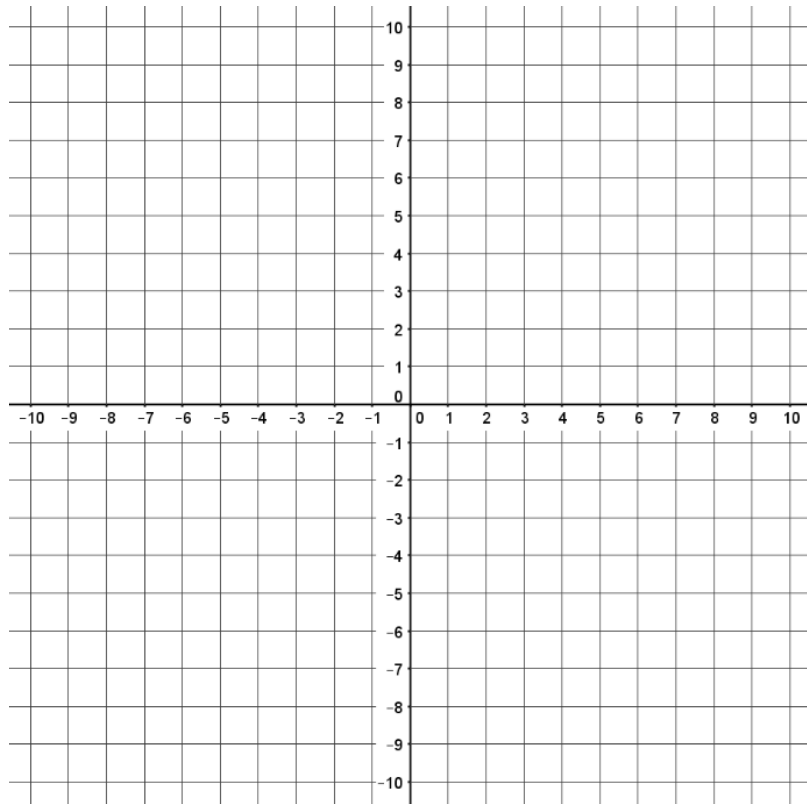
Unit 3 Lesson 09: Differentiate Implicitly

Lesson Objectives

- Differentiate equations that aren't solved for y.

6. A circle with radius 4 and center at (2, -1) has an equation of $(x - 2)^2 + (y + 1)^2 = 16$.

    a. Determine the slope of the tangent line at (-0.828, 1.828), and check by graphing.



    b. Determine the slope of the tangent line at (5.578, 0.789), and check by graphing.

    c. At what points will the tangent lines be horizontal?

    d. At what points will the tangent lines be vertical?

7. Find dy/dx of the equation $x^3 + y^3 = 18xy$ and determine the slope of the tangent line at (2, 1)
   Ans. dy/dx = (-2/11)

8. Find dy/dx of the equation $y^2 = \frac{x-1}{x+1}$ and determine the slope of the tangent line at (3, 0.25)
   Ans. dy/dx = 0.25

9. Find dy/dx of the equation x = sin y and determine the slope of the tangent line at (1, π/3)
   Ans. dy/dx = 2

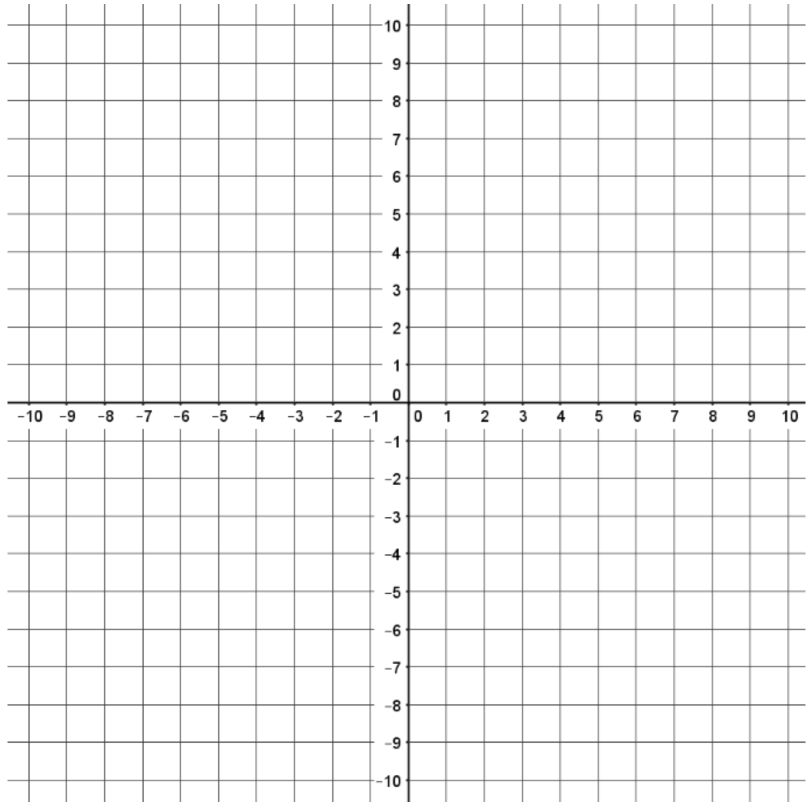
10. Find dy/dx of the equation x + sin y = xy and determine the slope of the tangent line at (2, 0)
    Ans. dy/dx = 1

~~~Homework U3L9~~~ 4.2: 1, 55a,b,c

~~~Classwork U3L9~~~

1. A circle with radius 5 and center at (-3, 2) has an equation of $(x + 3)^2 + (y - 2)^2 = 25$.

   a. Determine the slope of the tangent line at (-6.536, -1.536), and check by graphing.



   b. Determine the slope of the tangent line at (-5.236, 6.472), and check by graphing.

   c. At what points will the tangent lines be horizontal?

   d. At what points will the tangent lines be vertical?

2. Find dy/dx of the equation $x^2 + xy - y^2 = 3$ and determine the slope of the tangent line at (1, 1)
   Ans. dy/dx = 3

3. Find dy/dx of the equation $x^2 = \frac{x - y}{x + y}$ and determine the slope of the tangent line at (3, 2)
   Ans. dy/dx = -24.33

4. Find dy/dx of the equation x = cos y and determine the slope of the tangent line at (1, π/6)
   Ans. dy/dx = -2

5. Find dy/dx of the equation $x^{2/3} - y^{2/3} = 1$ and determine the slope of the tangent line at (64, 27)
   Ans. dy/dx = 0.75

Unit 3 Lesson 10: Determine Higher Order Derivatives

Lesson Objectives

- Differentiate more than once

Notations:

- The rate of change of position is _________________
- The rate of change of velocity is _________________
- Therefore, the rate of change of the rate of change of position is _________________

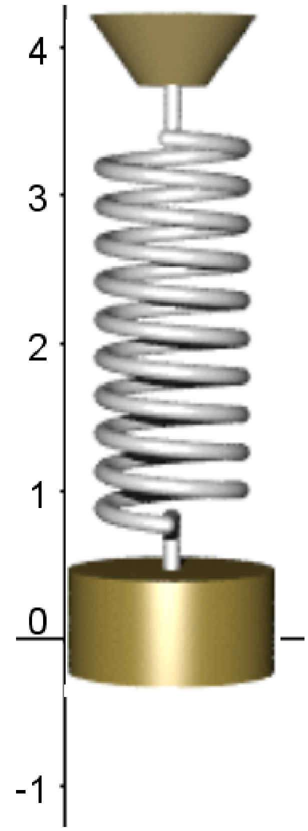
1. The height of a ball dropped from a 1000ft tall building can be found using the equation
   $y = (-16\text{ft/sec}^2)t^2 + 1000\text{ft}$.
   a. Find the velocity of the ball when it has been falling for 2 seconds.

   b. Find the velocity of the ball when it has been falling for 0 seconds.

   c. Find the acceleration of the ball when it has been falling for 2 seconds.

   d. Find the acceleration of the ball when it has been falling for 0 seconds.

2. A weight at y = 1ft is hanging peacefully on a spring, minding its own business. Alas, some wretched scoundrel comes and stretches it down 1 foot below the resting position, dooming the weight to an eternity of oscillation. At time
t = 0sec, the weight is released. The height of the weight could be given by the equation $f(t) = (-1\text{ft})\cos t + 1\text{ft}$



   a. What is the position of the weight at
      i. t = 0 seconds?
      ii. t = π seconds?
      iii. When is the first time the spring will return to its resting position?
      iv. When is the first time the spring will return to the position it was released from?
   b. What is the velocity of the weight at
      i. t = 0 seconds?
      ii. t = π/2 seconds?
      iii. t = π seconds?
      iv. t = 3π/2 seconds?
      v. t = 2π seconds?
   c. What is the acceleration of the weight at
      i. t = 0 seconds?
      ii. t = π/2 seconds?
      iii. t = π seconds?
      iv. t = 3π/2 seconds?
      v. t = 2π seconds?

~~~Homework U3L10~~~ 1. Find f ''(x) if $f(x) = 2x^4 + e^{2x}$ 2. Find f'''(x) if $f(x) = xe^x$ 3. Find y'' if $y = x^2\sin x$

1. Find $\frac{d^2y}{dx^2}$ if y = xlnx

2. Find f''(t) if f(t) = e^tcos$t$

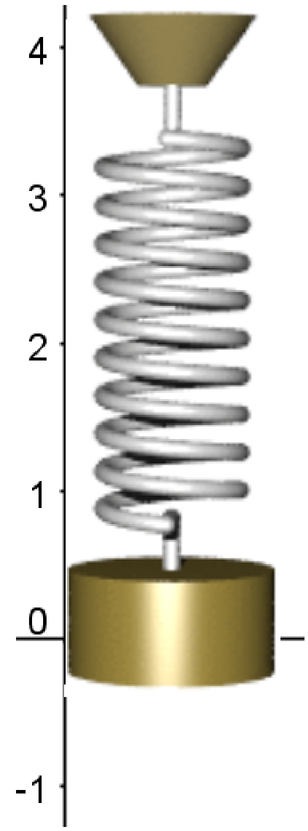
3. The height of a ball dropped from a 300m tall building can be found using the equation
   y = (-9.81m/sec^2)t^2 + 300m.
   a. Find the velocity of the ball when it has been falling for 2 seconds.

   b. Find the velocity of the ball when it has been falling for 0 seconds.

   c. Find the acceleration of the ball when it has been falling for 2 seconds.

   d. Find the acceleration of the ball when it has been falling for 0 seconds.

4. A weight at y = 0m is hanging peacefully on a spring, minding its own business. Alas, some wretched scoundrel comes and stretches it down 1 meter below the resting position, dooming the weight to an eternity of oscillation. At time
t = 0sec, the weight is released. The height of the weight could be given by the equation $f(t) = (-1\text{m})\cos(2\pi t)$

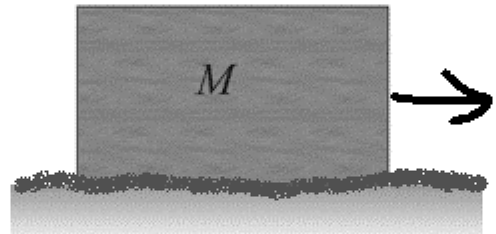


   a. What is the position of the weight at
      i. t = 0 seconds?
      ii. t = 0.5 seconds?
      iii. When is the first time the spring will return to its resting position?
      iv. When is the first time the spring will return to the position it was released from?
   b. What is the velocity of the weight at
      i. t = 0 seconds?
      ii. t = 0.25 seconds?
      iii. t = 0.5 seconds?
      iv. t = 0.75 seconds?
      v. t = 1 seconds?
   c. What is the acceleration of the weight at
      i. t = 0 seconds?
      ii. t = 0.25 seconds?
      iii. t = 0.5 seconds?
      iv. t = 0.75 seconds?
      v. t = 1 seconds?

4. At t = 0, some guy kicks a block that happened to be sitting on an oiled surface, sending it sliding at 40ft/sec in the positive x direction. Ah, good ol' Blocky, friend of physics teachers around the world. Anyways, Blocky slides along the oiled surface with equation $x = 40\ln(t+1)$ where x represents the horizontal distance traveled in feet, and t represents time in seconds. Answer the following questions.

   *extra credit: explain why each answer does or does not make physical sense

   a. What is the distance traveled when t = 0?



   b. What is the distance traveled when t = 3?

   c. What is the velocity of the block when t = 0?

   d. What is the velocity of the block when t = 3?

   e. What is the acceleration of the block when t = 0?

   f. What is the acceleration of the block when t = 3?