MATH 7

1.Square Roots and Cube Roots

- **Square Root**: Finding the square root is the inverse operation of squaring a number.
- For example, squaring 7 (7^2) gives 49; therefore, the square root of 49 ($\sqrt{49}$) is 7. However, keep in mind that $(-7)^2$ is also 49, so technically, both 7 and -7 are square roots of 49. The principal square root is the positive value.

- **Perfect Squares**: These are numbers that have integer square roots (e.g., 1, 4, 9, 16, 25…).

- **Non-Perfect Squares**: These numbers have irrational square roots (e.g., $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$). You'll often use a calculator or approximation methods for these.

- **Estimating Square Roots**: If you need to estimate a square root without a calculator, find the nearest perfect squares. For example, to estimate $\sqrt{20}$: $\sqrt{16} = 4$ and $\sqrt{25} = 5$, so $\sqrt{20}$ is between 4 and 5, closer to 4.

- **Cube Root:** Similar to square roots, but involves finding a number that, when multiplied by itself three times, gives the original number.

- **Perfect Cubes:** These are numbers with integer cube roots (e.g., 1, 8, 27, 64, 125…).

- **Non-Perfect Cubes:** These have irrational cube roots. Again, calculators or approximation methods are usually necessary.

- Example: $\sqrt[3]{27} = 3$ because 3 x 3 x 3 = 27. $\sqrt[3]{1000} = 10$.

2. Irrational Numbers

- Properties: Irrational numbers are real numbers, meaning they exist on the number line. They are non-terminating (their decimal representation goes on forever) and non-repeating (there's no pattern in the decimal digits).

- Examples:

- π (pi) ≈ 3.1415926535… (the ratio of a circle's circumference to its diameter)

- e (Euler's number) ≈ 2.71828… (used in calculus and compound interest calculations)

- √2 ≈ 1.41421356… (the length of the diagonal of a unit square)

- The golden ratio (φ) ≈ 1.6180339887… (found in nature and art)

- Operations: You can perform arithmetic operations (addition, subtraction, multiplication, division) with irrational numbers, but the result may or may not be irrational. For example, π + 2 is still irrational, but 2π/π = 2 (rational).

3. Conversion of Measurements

- **Metric System:** The beauty of the metric system is its consistency. All units are related by powers of 10.

- **Length:** kilometer (km), meter (m), centimeter (cm), millimeter (mm), micrometer (μm)

- **Mass**: kilogram (kg), gram (g), milligram (mg)

- **Volume**: liter (L), milliliter (mL), cubic centimeter (cc or cm^3) (Note: 1 mL = 1 cm^3)

- Example (kg to mg): Convert 0.5 kg to milligrams. 1 kg = 1000 g, and 1 g = 1000 mg. Therefore, 0.5 kg = 0.5 * 1000 * 1000 mg = 500,000 mg.

- **US Customary System**: Conversions are less straightforward because the relationships between units aren't based on powers of 10.

- **Length**: mile (mi), yard (yd), foot (ft), inch (in) (1 mi = 5280 ft, 1 yd = 3 ft, 1 ft = 12 in)

- **Weight**: ton (tn), pound (lb), ounce (oz) (1 tn = 2000 lb, 1 lb = 16 oz)

- **Volume**: gallon (gal), quart (qt), pint (pt), cup ©, fluid ounce (fl oz) (1 gal = 4 qt, 1 qt = 2 pt, 1 pt = 2 c, 1 c = 8 fl oz)

- Example: Convert 5 feet to yards. Since 3 feet = 1 yard, 5 feet = 5/3 yards ≈ 1.67 yards.

4. Unit Conversion (Medium): These often involve multiple steps. For example, converting kilometers per hour to meters per second requires converting kilometers to meters and hours to seconds.

5. Volume Calculations

- **Volume of a Cylinder:** The formula $V = \pi r^2 h$ is derived from the area of a circle (πr^2) multiplied by the height. Remember that 'r' is the radius (half the diameter).

- Example: A cylindrical water tank has a diameter of 2 meters and a height of 5 meters. Find its volume.

- **Radius** ® = diameter/2 = 2 m / 2 = 1 m

- **Volume** (V) = $\pi(1\text{ m})^2(5\text{ m}) \approx 15.7$ cubic meters

- **Volume of a Pyramid**: The formula $V = (1/3)Bh$ comes from the fact that a pyramid's volume is one-third the volume of a prism with the same base and height.

- Example: A square pyramid has a base of 6 cm x 6 cm and a height of 8 cm.

- Base area (B) = 6 cm x 6 cm = 36 sq cm

- Volume (V) = (1/3)(36 sq cm)(8 cm) = 96 cubic cm

6. Sets

- **Set Notation**: Sets are often represented using capital letters (A, B, C,…). Elements are listed within curly braces {} and separated by commas.

- Example: The set of even numbers less than 10: E = {2, 4, 6, 8}

- Example: The set of vowels: V = {a, e, I, o, u}

- Example: The empty set (a set with no elements): Ø or {}

- **Subsets**: If all elements of set A are also in set B, then A is a subset of B (written as $A \subset B$).

- Example: If A = {1, 2} and B = {1, 2, 3}, then $A \subset B$.

- Example: Every set is a subset of itself ($A \subset A$).

- **Sets and Real Numbers:** Real numbers can be elements of sets. You can define sets based on properties of real numbers (e.g., the set of all positive integers, the set of all real numbers between -1 and 1). This leads to concepts like intervals (open or closed intervals) which are used in calculus.

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