

Assignment No 1

MTH646 (Fall 2024)

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Question No.01

Consider the following discretised form of heat equation

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

Subject to the boundary and initial conditions:

$$w(0,t) = w_{0,j} = 0$$

$$w(1,t) = w_{10,j} = 0$$

$$w(x,t) = w_{i,0} = \sin \pi x_i$$

After finding h , calculate the values of $w_{1,1}$ and $w_{2,1}$.

Solution:

Given that,

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

Boundary conditions and initial conditions are following:

$$w(0,t) = w_{0,j} = 0$$

$$w(1,t) = w_{10,j} = 0$$

$$w(x,t) = w_{i,0} = \sin \pi x_i$$

Firstly, we find the value of h ,

$$h = \frac{b-a}{n}$$

$$h = \frac{1-0}{10} = 0.1$$

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j} \dots\dots(1)$$

Initial condition is:

$$w(x, t) = w_{i,0} = \sin \pi x_i$$

Fix $t = 0$:

$$w(1, 0) = w_{1,0} = \sin \pi (0.1) = 0.3090$$

$$w(2, 0) = w_{2,0} = \sin \pi (0.2) = 0.5877$$

$$w(3, 0) = w_{3,0} = \sin \pi (0.3) = 0.8090$$

For $i = 1$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{1,1} = w_{0,0} - w_{1,0} + w_{2,0}$$

$$w_{1,1} = 0 - 0.3090 + 0.5877 = 0.2787$$

For $i = 2$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{2,1} = w_{1,0} - w_{2,0} + w_{3,0}$$

$$w_{2,1} = 0.3090 - 0.5877 + 0.8090 = 0.5302$$

Question No.02:

Consider the following discretised form of heat equation

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

Subject to the boundary and initial conditions:

$$w(0,t) = w_{0,j} = 0$$

$$w(1,t) = w_{10,j} = 0$$

$$w(x,t) = w_{i,0} = \sin \pi x_i$$

After finding h, calculate the values of $w_{8,1}$ and $w_{9,1}$.

Solution:

Boundary conditions and initial conditions are following:

$$w(0,t) = w_{0,j} = 0$$

$$w(1,t) = w_{10,j} = 0$$

$$w(x,t) = w_{i,0} = \sin \pi x_i$$

Firstly, we find the value of h ,

$$h = \frac{b-a}{n}$$

$$h = \frac{1-0}{10} = 0.1$$

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j} \dots\dots(1)$$

Initial condition is:

$$w(x,t) = w_{i,0} = \sin \pi x_i$$

Fix $t = 0$:

$$w(0,0) = w_{0,0} = \sin \pi(0) = 0$$

$$w(1,0) = w_{1,0} = \sin \pi(0.1) = 0.3090$$

$$w(2,0) = w_{2,0} = \sin \pi(0.2) = 0.5878$$

$$w(3,0) = w_{3,0} = \sin \pi(0.3) = 0.8090$$

$$w(4,0) = w_{4,0} = \sin \pi(0.4) = 0.9511$$

$$w(5,0) = w_{5,0} = \sin \pi(0.5) = 1$$

$$w(6,0) = w_{6,0} = \sin \pi(0.6) = 0.9511$$

$$w(7,0) = w_{7,0} = \sin \pi(0.7) = 0.8090$$

$$w(8,0) = w_{8,0} = \sin \pi(0.8) = 0.5878$$

$$w(9,0) = w_{9,0} = \sin \pi(0.9) = 0.3090$$

$$w(10,0) = w_{10,0} = \sin \pi(1) = 0$$

For $i = 1$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{1,1} = w_{0,0} - w_{1,0} + w_{2,0}$$

$$w_{1,1} = 0 - 0.3090 + 0.5877 = 0.2787$$

For $i = 2$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{2,1} = w_{1,0} - w_{2,0} + w_{3,0}$$

$$w_{2,1} = 0.3090 - 0.5877 + 0.8090 = 0.5302$$

For $i = 3$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{3,1} = w_{2,0} - w_{3,0} + w_{4,0}$$

$$w_{3,1} = 0.5878 - 0.8090 + 0.9511 = 0.7299$$

For $i = 4$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{4,1} = w_{3,0} - w_{4,0} + w_{5,0}$$

$$w_{4,1} = 0.8090 - 0.9511 + 1 = 0.8579$$

For $i = 5$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{5,1} = w_{4,0} - w_{5,0} + w_{6,0}$$

$$w_{5,1} = 0.9511 - 1 + 0.9511 = 0.9022$$

For $i = 6$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{6,1} = w_{5,0} - w_{6,0} + w_{7,0}$$

$$w_{6,1} = 1 - 0.9511 + 0.8090 = 0.8579$$

For $i = 7$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{7,1} = w_{6,0} - w_{7,0} + w_{8,0}$$

$$w_{7,1} = 0.9511 - 0.8090 + 0.5878 = 0.7299$$

For $i = 8$ and $j = 0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{8,1} = w_{7,0} - w_{8,0} + w_{9,0}$$

$$w_{8,1} = 0.8090 - 0.5878 + 0.3090 = 0.5302$$

For $i=9$ and $j=0$:

$$w_{i,j+1} = w_{i-1,j} - w_{i,j} + w_{i+1,j}$$

$$w_{9,1} = w_{8,0} - w_{9,0} + w_{10,0}$$

$$w_{9,1} = 0.5878 - 0.3090 + 0 = 0.2788$$