Advanced Probability Class Notes

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This course focuses on probability concentration bounds, with applications to statistical learning and networks. This is primarily a "tools" class focusing on mathematical methods and proofs. The tools developed in this class are useful in a variety of domains, including statistical machine learning, communication/social networks analysis, and epidemics/influence propagation.

The material covered in these notes are drawn from a number of papers and books. The primary references from which material is drawn heavily are:

1. Concentration of Measure for the Analysis of Randomized Algorithms, D. Dubhashi and A. Panconesi, Cambridge University Press, 2009.
2. Concentration Inequalities - A Nonasymptotic Theory of Independence, S. Boucheron, G. Lugosi and P. Massart, Oxford University Press, 2013. See also Concentration of measure inequalities, G. Lugosi, Lecture notes, 2009: http://www.econ.upf.es/~lugosi/anu.pdf
3. A Probabilistic Theory of Pattern Recognition, L. Devroye, L. Gyorfi, and G. Lugosi. Springer 1996.
4. Large Deviations Techniques and Applications, A. Dembo and O. Zeitouni, Springer, 2nd ed., 1998.
5. Concentration Inequalities and Model Selection, P. Massart, Lecture Notes in Mathematics 1896, Springer, 2007.
6. Stochastic Approximation: A Dynamical Systems Viewpoint, V. Borkar, Cambridge University Press, 2008.
7. Markov Chains: Gibbs Fields, Monte Carlo Simulation and Queues, P. Bremaud, Springer, 1998.

Many other books/papers are used; the detailed references are available in the individual lecture notes.

The topics covered are:

1. **Overview and Concentration Techniques** – Chernoff, Hoeffding, Bernstein, Negative association
2. **Martingale Methods** – Doob's martingale, bounded difference and bounded variance methods (Azuma-Hoeffding / McDiarmid's inequality), Median concentration, Blowing up lemma

3. **Markov Chains** - Perron Frobenius theorem, MGF bounds through perturbation theory of linear operators, Concentrations for reversible and non-reversible Markov Chains
4. **Hypercontractivity** - Hypercontractivity bounds, Concentrations for low-degree polynomial functions of Boolean and Gaussian random variables
5. **Talagrand's Inequality and Applications** – Talagrand's inequality for Hamming distance, configuration functions, Applications to Euclidean Traveling Salesman Problem (ETSP), Spanning Trees
6. **Transportation Inequalities** – Marton's method (Tensorization of Pinsker's inequality), Quadratic transportation inequality
7. **Entropy Method** – Log-Sobolev inequalities, applications, Proof of Talagrand's inequality
8. **Stein's Method** – Stein's Lemma, Berry-Esseen theorem, CLT for dependent random variables, Size-bias and zero-bias coupling, Stein-Chen method for Poisson approximations, Law of small numbers, Applications to random graphs
9. **Mean Field Limits** – Kurtz's theorem, Exchange of time and space limits for many-particle systems, Exchangeability and connection to propagation of chaos

The notes are available at the Dropbox link below. We emphasize that the material in the notes are heavily drawn from the references above and many papers (cited in the notes).

Notes: https://www.dropbox.com/sh/y6fth1sdgcynn0k/AACkxJVoFy-Sq9zOTmrGDdzha?dl=0

Caveat: These notes likely have several typos (and possibly other errors). I would greatly appreciate it if you could email me at sanjay.shakkottai@utexas.edu with any corrections/typos.