Testing Binary Star Formation Models Using N-Body Simulations

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Introduction

In our Milky Way galaxy alone there are around 100 billion stars. It is clear to the naked eye that stars come in a range of sizes, masses, and brightness. Additionally, about 50% of the stars visible from Earth come in pairs, orbiting each other's center of mass as a binary star system. The separations of these binary systems are typically 100-1000 AU, where an AU is the average distance between the Earth and the Sun. It is now understood that these properties of stars are determined in the early years of a star's life.

There are two leading ideas for how binary star systems form. The core collapse model posits that binarity is determined simultaneously with the birth of the stars; i.e., star pairs form together. The cluster model suggests instead that as stars roam around together as a cluster, star-star interactions and motion through their natal cloud pair stars together. This project explored the feasibility of the cluster model through the use of numerical simulations. In particular, we simulated several 1000-star clusters for 20 million years. At the end of the simulations, we assessed how well the simulation reproduced (1) the number of binary star systems, and (2) the distribution of separations between the stars in binaries, as compared to observations. **If the simulations are able to reproduce the kinds of binaries we see in the Milky Way, this lends support to the cluster model as a viable avenue for binary formation.**

Method

If we hope to study the 3d gravitational interactions within the cluster across its entire life span, numerical simulations are necessary. These objects are well suited for N-Body-type simulations, which solve Newton's equations of motion for a collection of particles, in our case stars subject to gravitational tugs and pulls with each other. These simulations of star clusters, which can be completed in a number of days to weeks, yield rich 7 dimensional data sets providing 3d positions, 3d velocities, and masses for each star, all as a function of time.

We used the software package *Rebound*, written in Python and C++, to evolve gravitationally interacting systems. *Rebound* only includes gravitational forces. For some of our simulations we wished to model the stars moving through their remnant gas cloud, which necessarily introduces additional physics. This includes the ability for stars to gravitationally collect mass, modeled as Bondi-Hoyle accretion (Bondi 1952):

$$\dot{m} = \frac{dm}{dt} = \dot{M}_{BH}(\rho, \vec{v}) = \frac{4\pi G^2 m^2 \rho}{c^3} / (1 + (v/c_s)^2)^{3/2},$$

where G is Newton's gravitational constant, m is the mass of the star, is the local gas density, v is the speed of the star relative to the gas, and c_s is the isothermal sound speed of gas at 10 Kelvin. The stars also experience a drag force, which we model based off Lee & Stahler (2011):

$$\vec{F} = -\dot{M}_{BH}(\rho, \vec{v})\, \vec{v}$$

While we do not directly simulate the gas of the gas cloud, we model its presence as a spherical Plummer sphere:

$$\rho(r) = \frac{3M_{gas}}{4\pi a^3}(1 + r^2/a^2)^{-5/2},$$

where r is the radial distance from the center of the cloud. This assumes a total mass in the gas $M_{gas} = 10^4\ M_{sun}$ and characteristic length a = 3.25 light-years. These values are taken from typical nebula in the Milky Way. The gas also introduces an additional gravitational force toward the center.

Initial Conditions

Initial positions of these stars are randomly positioned within a spherical volume of radius 3.25 light-years with an initial velocity randomly oriented and equal to 1.466 km/s. These values are based on observations of real star clusters. Our parameters to explore include (1) the total number of stars, (2) the initial mass distribution of the stars, and (3) the presence or lack thereof of the gas cloud. The physics of each simulation performed is outlined in the following table.

Sim	Gravity		
1	✔		
2	✔	✔	
3	✔		✔
4	✔	✔	✔

Each simulation is run for 20 million years , and to aid in comparing simulations, each simulation using the same random number seed to generate the initial positions and velocities. To assess the sensitivity of the results on the initial conditions, we ran 4 versions of Sim1 using different seeds. While the simulations are not identical, the global averages of relevant quantities should be statistically similar. In particular, we looked at the fraction of stars paired as binaries at the end of each simulation, shown in the following table.

Gravity Simulations	# of Binaries at the End Of The Simulation
1	206
2	255
3	146
4	246

The average and standard deviation for this suite is 210 +- 40. If the mean value is representative of the typical number of expected binaries, three of four simulations are consistent with this value, with Sim3 being just shy of one standard deviation from the mean. While more simulations would be helpful to test this sensitivity, this nonetheless supports the claim that initial conditions do not significantly impact our results.

Binaries

The definition of a binary system requires that the total energy (kinetic + gravitational potential energy), when viewed in the center of mass frame of the two stars, is negative:

$$E = \frac{1}{2}\mu v_{rel}^2 - \frac{G\mu M_{tot}}{r_{rel}} < 0$$

Where is the reduced mass parameter

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1 m_2}{M_{tot}},$$

And the velocity and position are the relative values between the two stars. The binaries can be identified for every simulation output (every 10,000 years), giving information about binaries at the end of the simulation as well as a function of time.

To match observations, the binaries at the end of the simulation should pass two tests: Test 1 is that at least 50% of the stars should be binaries, and Test 2 being that the sizes of the orbits should be generally in the range of 100 to 1000 AU.

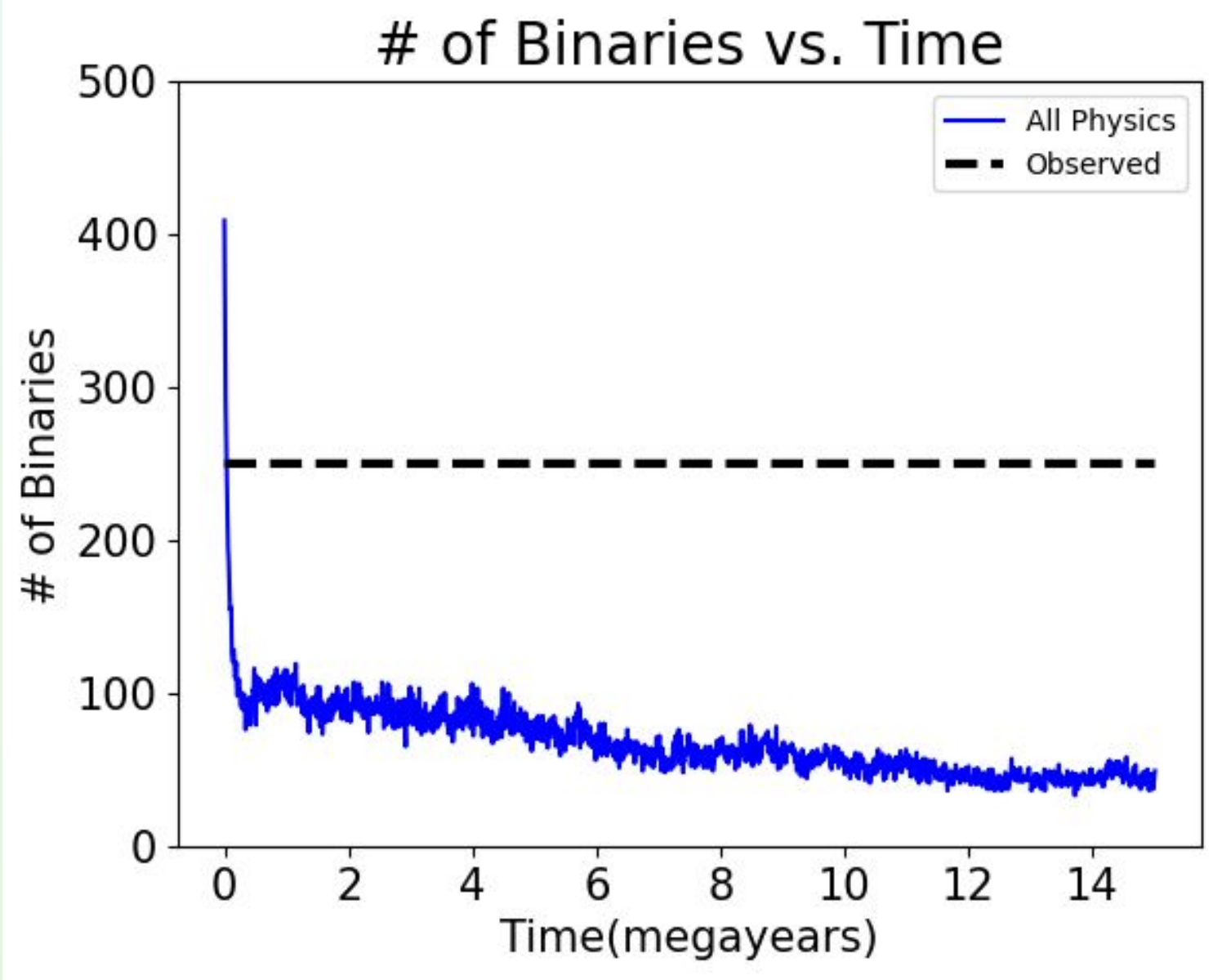

Figure 1

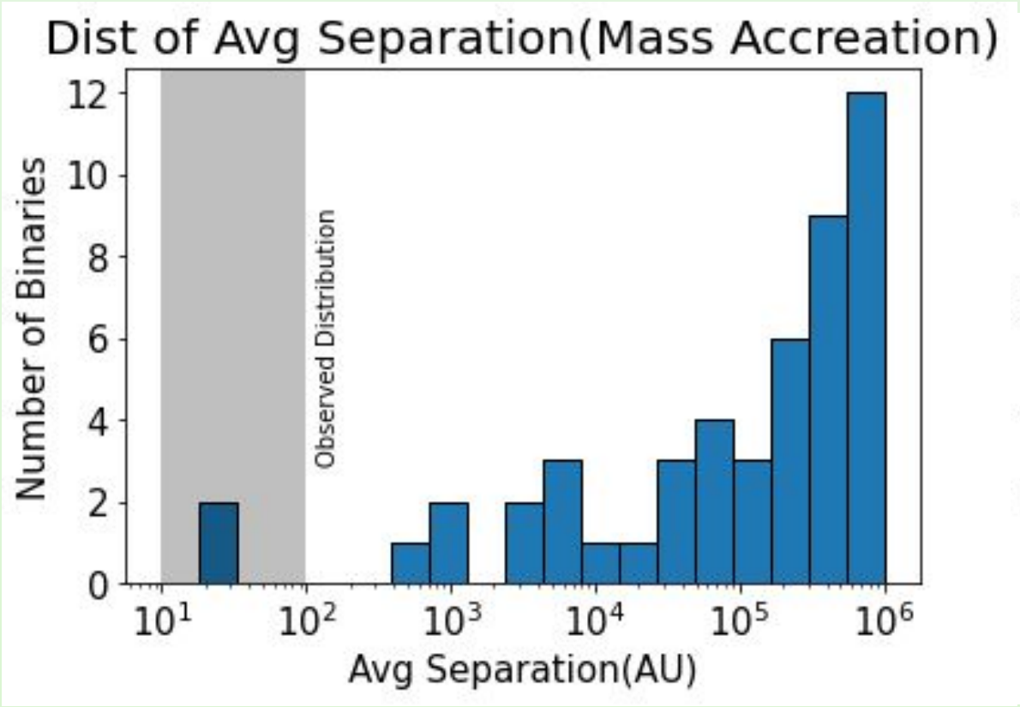

Figure 2

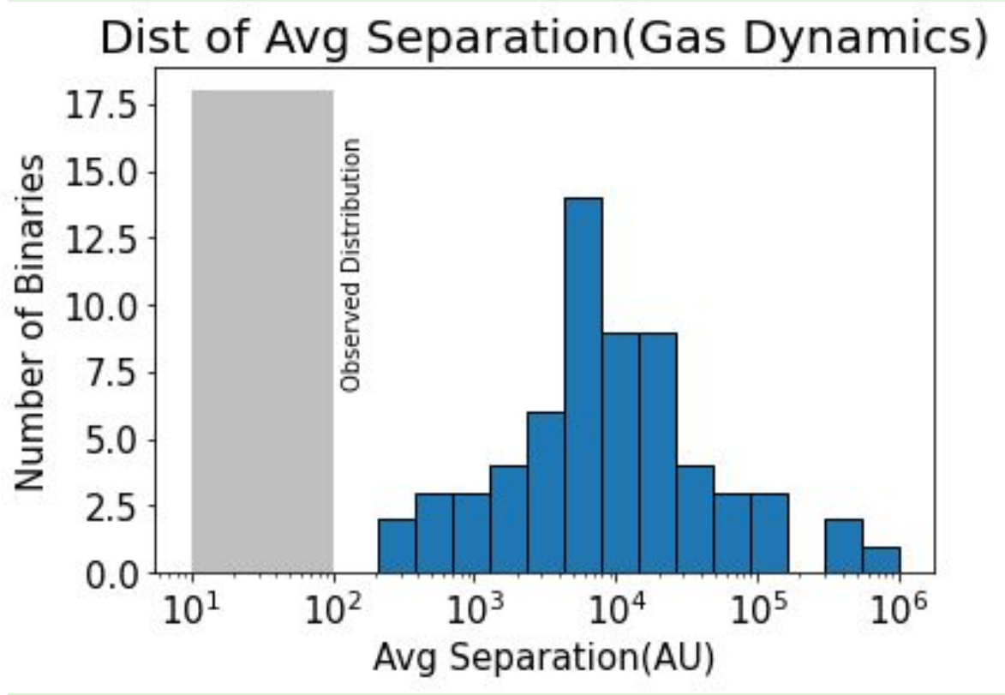

Figure 3

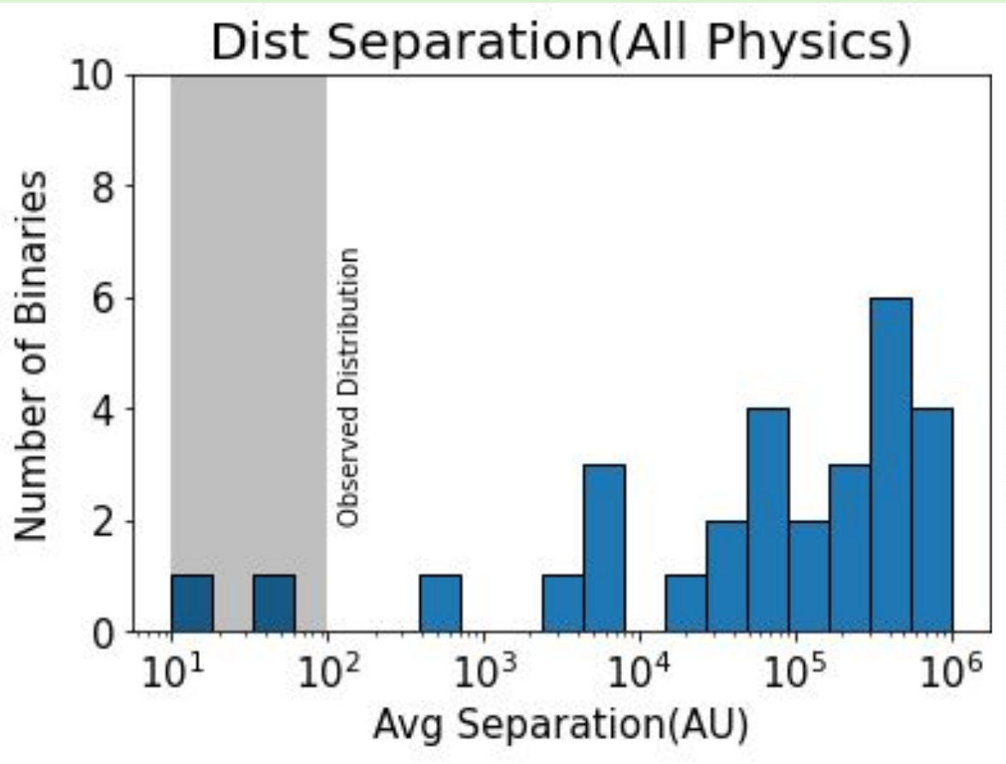

Figure 4

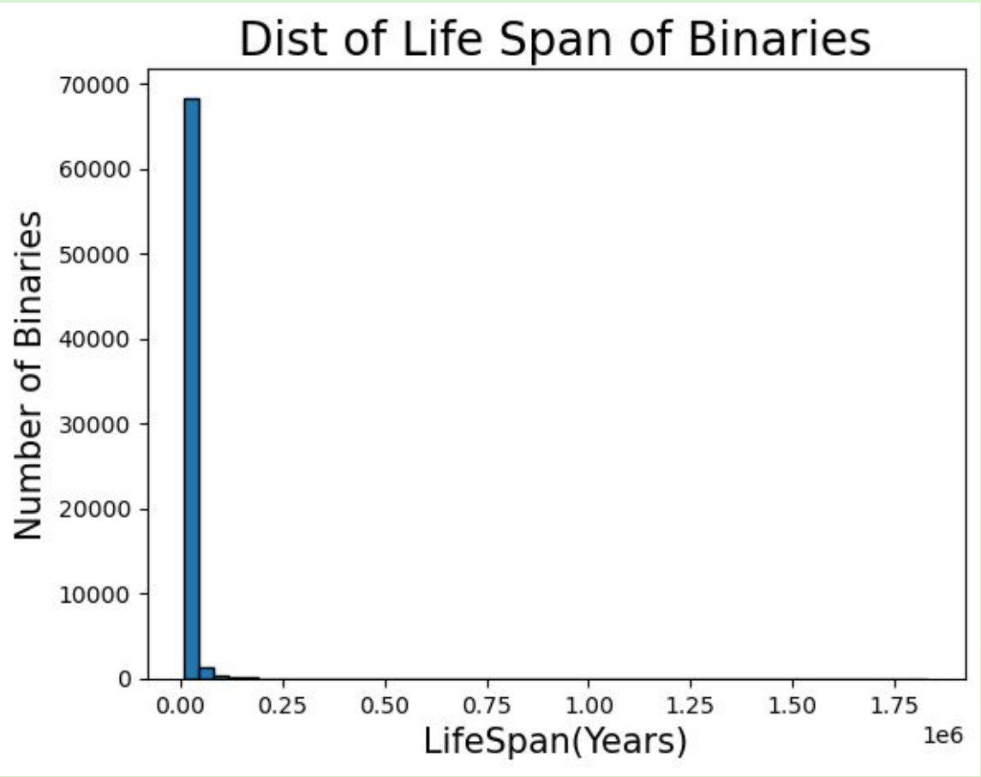

Figure 5

Ultimately the binaries failed both tests. Figure 1 shows the number of binaries as a function of time across the simulation. To pass the first test the number of binaries should be at or above this dashed line and it falls short failing the test. For test 2 Figure 4 shows the distribution of separations at the end of the simulation. To pass the test most of the data should lie in the gray box, however, the vast majority of binaries have separation that are about a thousand times too big. By including a gas drag force, however, binary orbits should ultimately shrink with time eventually passing this test. However, seen with Figure 5, the binaries are short lived, existing, on average, for about 100,000 years before becoming unbound or switching companions. Therefore a given binary does not last long enough to decay, rather, the stars more likely separate entirely or restart as a new wide orbit binary with other stars. So more simulation time should not change this plot.

Conclusion/Future Work:

While our results did not match observations, it does not mean our results are bad. Rather, it tells us that this model alone cannot be solely responsible for the binaries we see. Several additional simulations are underway which will further test this model in star clusters with significantly more or fewer stars.

Acknowledgements:

I would like to thank my mentor, Dr. Lee, my colleague, Dhruva Navin Chander, as well as the SRP program for their continued support. I would also like to thank the School of Science & John Macken for their contribution.

References:

Bondi, H. 1952. MNRAS, 112, 195.
Lee, A.T. & Stahler, S.W. 2011. MNRAS, 416, 3177.
Plummer, H.C. 1911. MNRAS, 71, 460.